The Origin and Meaning of 3-Dimensionality

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Abstract: 3-dimensionality is identified as one of the most profound and fundamental concepts in physics. With its origin in ideas of anticommutativity, which are antecedent to the concept of number, it is responsible for all discreteness in physical systems, and in particular for quantization. It is responsible for symmetry breaking between the forces, for many significant aspects of particle structure, and for most of the manifestations of the number 3 that are considered fundamental in physics. It is responsible for the selection of the fundamental parameters that we use in the most basic physical explanations, and for their special properties, and the Dirac equation is specially structured to accommodate it. No other dimensionality, not even that of '4-dimensional' space-time, has any fundamental physical significance, a fact which has extremely profound consequences for a unified theory.

1 Discreteness and dimensionality

String theorists regularly talk of 10 and 11 dimensions. Special relativity uses a 4-dimensional space-time. Kaluza-Klein theory introduces a fifth dimension to general relativity to account for the electromagnetic field. All this is done on the basis that the origin and meaning of dimensionality in nature are matters still to be decided, that, *a priori*, no particular number of dimensions is more likely than any other, that no number associated with dimensionality can be privileged, and that the actual number of dimensions is still negotiable.

Yet 3-dimensionality is very special. It has a mathematical as well as physical validity, which should make us wary of cavalierly expanding the number of dimensions in our system to meet the immediate needs of defining a physically inclusive theory. And the number 3 appears everywhere in fundamental physical contexts – 3 dimensions of space, 3 nongravitational interactions, 3 fundamental symmetries (*C*, *P* and *T*), 3 conserved dynamical quantities (momentum, angular momentum and energy), 3 quarks in a baryon, 3 generations of fermions. Could these be in any way related, and could the explanation of their common 'threeness' somehow lead to a deeper explanation of physical 'reality' than is apparent from the more complex attempts at explanation represented by string theory? What could be special about the number 3 which could unite these apparently disparate manifestations of its application?

To answer this, I need to draw upon an idea which I have long held and often written about. (Rowlands, 1983, 1991, 1999, 2001) This is the idea that the most fundamental concepts in physics are the parameters space and time, and the sources of the four fundamental interactions, namely mass (or mass-energy) and three types of ‘charge’ (electromagnetic, strong and weak), which it is convenient to describe as orthogonal dimensions in a ‘charge space’. Seemingly, these parameters are symmetric according to a Klein-4 scheme, with the following properties and exactly opposite ‘antiproperties’:

mass	conserved	real	continuous nondimensional
time	nonconserved	imaginary	continuous nondimensional
charge	conserved	imaginary	discrete dimensional
space	nonconserved	real	discrete dimensional

A remarkable aspect of this symmetry lies in the last column, where there are *two* properties and *two* antiproperties, which, if the symmetry is exact, must be linked. We are obliged, it would seem, to suppose that discreteness and dimensionality are intrinsically related properties. In addition, though it is apparent that discreteness and continuity can be considered as a genuinely opposite pairing of property and antiproperty, it is not quite so obvious that the same applies to (3)-dimensionality and nondimensionality, or, as it is sometimes called, 1-dimensionality.

However, ‘1-dimensional’ quantities are not really dimensional at all, and it is relatively easy to see why an absolutely continuous quantity cannot be dimensional. Dimensionality requires an origin, a cross-over point or zero position, that is a distinct discontinuity of some kind, which is of course incompatible with the kind of absolute continuity which makes time irreversible and mass-energy unipolar and ubiquitous in the form of the vacuum or Higgs field.

If the linkage here seems relatively obvious, it is far from obvious that discrete quantities must be dimensional, and specifically 3-dimensional at that. However, each of the two known dimensional quantities seems to supply half of the required explanation. Thus, the discreteness of space is associated with its use as the unique channel of physical measurement because the *nonconserved* nature of space means that its discreteness can be endlessly restructured. Measurement would, of course, be impossible in one dimension. A continuous line would offer no possibility of measurement unless it could be drawn in a 2-dimensional space with the other dimension providing the marking off of the zero points or origins. At the same time, the imaginary nature of charge would appear to imply that any dimensionality associated with this quantity must be 3-dimensionality, as required by the algebra of quaternions.

The link between discreteness and 3-dimensionality would appear to be a result of the other properties associated with the parameters which we have defined as discrete, and there appears to be no direct route to be found leading from discreteness itself to dimensionality. However, this is not the case with the reverse process, and it is in fact possible to show that *dimensionality is really the primary property*, and that all discreteness in nature results from dimensionality, and that *only 3-dimensional quantities* are discrete.

2 The consequences of zero totality

To find the route from dimensionality to discreteness, we again refer to the table of properties for the fundamental parameters, and observe that, in the conceptual sense, it represents a *zero totality*, every property in a parameter being negated by the corresponding antiproperty in another. Remarkably, it is from this perceived need to preserve a zero totality in nature that 3-dimensionality ultimately springs. If we suppose that the only logical condition which incorporates no special assumptions is a zero totality, we may assume that any attempted deviation from this state will automatically generate its own zeroing mechanism, exactly as we observe in the physical world when we conserve momentum or angular momentum.

Let us suppose that we describe deviations from zero by a nonunique term $\mathfrak{R}$, which remains simply unspecified and undefined, and which can only be examined in relation to itself in such a way that it forces an attempt at recovering the original zero totality. The immediate outcome of the ‘self-examination’ $(\mathfrak{R}) \times (\mathfrak{R}) = (\mathfrak{R})$ will be a *conjugate*, or zero-producing, term, which we may represent by $-\mathfrak{R}$, without making any assumptions about a specific mathematical meaning for the symbol. Again, without making mathematical assumptions, we can represent the process in the form:

$$(\mathfrak{R}) \times (\mathfrak{R}) = (\mathfrak{R}) \to (\mathfrak{R}, -\mathfrak{R})$$

Let us describe objects of the form ($\mathfrak{R}$) and ($\mathfrak{R}$, $-\mathfrak{R}$) as ‘alphabets’, and any representations of components as ‘subalphabets’ (Rowlands and Diaz, 2002). Then we may suppose that any examination of an alphabet in relation to a subalphabet will produce the original alphabet in the same way as $(\mathfrak{R}) \times (\mathfrak{R}) = (\mathfrak{R})$. So we have, for example:

$$(\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}) = (\mathfrak{R}, -\mathfrak{R}) \qquad \text{and} \qquad (-\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}) = (\mathfrak{R}, -\mathfrak{R}) \ .$$

We may assume, however, that the process of producing a conjugate is no more unique than the symbol and that the same will apply to the new symbol. So examining this in relation to itself will produce a new conjugated alphabet, for example:

$$(\mathfrak{R}, -\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}) = (\mathfrak{R}, -\mathfrak{R}) \to (\mathfrak{R}, -\mathfrak{R}, ₵, -₵)$$

where

$$(\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}, ₵, -₵) = (\mathfrak{R}, -\mathfrak{R}, ₵, -₵)$$
$$(-\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}, ₵, -₵) = (\mathfrak{R}, -\mathfrak{R}, ₵, -₵)$$
$$(\mathfrak{R}, -\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}, ₵, -₵) = (\mathfrak{R}, -\mathfrak{R}, ₵, -₵)$$
$$(₵, -₵) \times (\mathfrak{R}, -\mathfrak{R}, ₵, -₵) = (\mathfrak{R}, -\mathfrak{R}, ₵, -₵) \text{ , etc.}$$

From such expressions, we derive also the results of the ‘actions’ of subalphabets upon each other. For example:

$$(-\mathfrak{R}) \times (-\mathfrak{R}) = (\mathfrak{R})$$
$$(\mathfrak{R}) \times (₵) = (₵)$$
$$(₵) \times (₵) = (-\mathfrak{R})$$
$$(₵) \times (-₵) = (\mathfrak{R})$$

Further conjugated alphabets will be constructed with appropriate subalphabets so that the correct rules will automatically apply.

$$(\mathfrak{R}, -\mathfrak{R}, ₵, -₵) \times (\mathfrak{R}, -\mathfrak{R}, ₵, -₵) = (\mathfrak{R}, -\mathfrak{R}, ₵, -₵, ₵₵', -₵', ₵₵', -₵₵') \ .$$

In effect, we will generate a series of ₵-type terms, ₵, ₵', ₵'', ₵''', etc., and their ‘actions’ upon each other, such as ₵₵'; and the ‘self-action’ of the ₵-type terms will always result in ($-\mathfrak{R}$). The series may be imagined as continuing to infinity, and generating itself in the supervenient, not temporal, sense, with all the terms existing at once. If, however, we take any pair of terms in the ₵ series, say *X* and *Y*, then *either*

$$(XY) \times (XY) = (-\mathfrak{R}) \qquad \textit{or} \qquad (XY) \times (XY) = (\mathfrak{R}) \ .$$

These may be described as the respective anticommutative and commutative options. The significant point here is that the anticommutative option can be used only once. For any given *X*, there is only a single *Y* and a single *XY*. *X*, *Y* and *XY* form a closed cycle. This is exactly what we mean by 3-dimensionality: it is the direct manifestation of closure through anticommutativity, and has no other fundamental significance. On the other hand, the commutative option remains open to infinity with an unlimited number of *Y* terms for any given *X*.

Clearly, there are an infinite set of available options within this alphabet-generating mechanism, but, going for maximum efficiency, i.e. the minimum generation of rules, we may set the default condition at the anticommutative option as the automatic choice whenever this is available. We then produce a regular sequence of closed finite-dimensional systems taking us to infinity, which has exactly the same form as the set of finite integers in conventional enumeration, and can be used exactly for this purpose, occurring even in a ready-made binary form. We create numbers at the same time as we create finite (3-)dimensionality, exactly as physics seems to suggest that we must.

3 The alphabet becomes algebra

With the concept of numbers established, we can now *choose* to interpret $\mathfrak{R}$, which is not itself defined in terms of finite enumeration, as the Cantorian or non-denumerable set of real numbers and the ₵-series as an infinite set of complex forms, whose real 'magnitudes', when closed, are represented by the constructible real numbers of Robinson's non-standard analysis or Skolem's non-standard arithmetic, and by the Cantorian reals when open. The – sign can be seen as referring to arithmetic or algebraic negation and × as arithmetic or algebraic multiplication. These definitions do not retrospectively limit the generality of the argument in the preceding section to a mathematical one, and only apply where we choose an option which introduces the counting of discrete numbers. We can now define counting units within $\mathfrak{R}$, ₵, ₵′, ₵″, ₵‴, ... as, say, 1, $\boldsymbol{i}_1$, $\boldsymbol{j}_1$, $\boldsymbol{i}_2$, $\boldsymbol{j}_2$,..., in which $\boldsymbol{i}_n$, $\boldsymbol{j}_n$, $\boldsymbol{i}_n \boldsymbol{j}_n = \boldsymbol{k}_n$, and so on, are independent sets of *quaternions*, following the usual rules determined by anticommutativity:

$$\boldsymbol{i}_n \boldsymbol{j}_n = -\boldsymbol{j}_n \boldsymbol{i}_n = \boldsymbol{k}_n$$
$$\boldsymbol{j}_n \boldsymbol{k}_n = -\boldsymbol{k}_n \boldsymbol{j}_n = \boldsymbol{i}_n$$
$$\boldsymbol{k}_n \boldsymbol{i}_n = -\boldsymbol{i}_n \boldsymbol{k}_n = \boldsymbol{j}_n$$
$$\boldsymbol{i}_n^2 = \boldsymbol{j}_n^2 = \boldsymbol{k}_n^2 = \boldsymbol{i}_n \boldsymbol{j}_n \boldsymbol{k}_n = -1 \quad .$$

All other products, however, follow the rules of commutativity. For example, when $m \neq n$,

$$\boldsymbol{i}_m \boldsymbol{i}_n = \boldsymbol{i}_n \boldsymbol{i}_m$$
$$\boldsymbol{i}_m \boldsymbol{j}_n = \boldsymbol{j}_n \boldsymbol{i}_m$$
$$\boldsymbol{j}_m \boldsymbol{j}_n = \boldsymbol{j}_n \boldsymbol{j}_m$$

and

$$(\boldsymbol{i}_m \boldsymbol{i}_n)(\boldsymbol{i}_m \boldsymbol{i}_n) = 1$$
$$(\boldsymbol{i}_m \boldsymbol{j}_n)(\boldsymbol{i}_m \boldsymbol{j}_n) = 1$$
$$(\boldsymbol{j}_m \boldsymbol{j}_n)(\boldsymbol{j}_m \boldsymbol{j}_n) = 1$$

even though

$$\boldsymbol{i}_m^2 = \boldsymbol{i}_n^2 = \boldsymbol{j}_m^2 = \boldsymbol{j}_n^2 = -1 \quad .$$

We can further interpret this algebraic series as a dualistic doubling of terms in each conjugation process, so that the series:

order 2	$(1, -1)$
order 4	$(1, -1) \times (1, \boldsymbol{i}_1)$
order 8	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$
order 16	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$
order 32	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$
order 64	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$,

generates the terms:

order 2	± 1
order 4	$\pm 1, \pm \boldsymbol{i}_1$
order 8	$\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1$
order 16	$\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
order 32	$\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
	$\pm \boldsymbol{j}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
order 64	$\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
	$\pm \boldsymbol{j}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
	$\pm \boldsymbol{i}_3, \pm \boldsymbol{i}_3\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
	$\pm \boldsymbol{i}_3\boldsymbol{j}_2, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1,$
	$\pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$

We recognise the algebraic groups in this series as those of:

order 2	real scalars
order 4	complex scalars (real scalars plus pseudoscalars)
order 8	quaternions
order 16	complex quaternions or multivariate 4-vectors
order 32	double quaternions
order 64	complex double quaternions or multivariate vector quaternions

Another way to look at the series is in terms of an endless succession of just three processes: conjugation (introducing opposite algebraic signs), complexification (multiplying by a single imaginary term); dimensionalization (multiplying by the imaginary term which will complete the quaternion set):

order 2	conjugation	$\times (1, -1)$
order 4	complexification	$\times (1, \boldsymbol{i}_1)$
order 8	dimensionalization	$\times (1, \boldsymbol{j}_1)$
order 16	complexification	$\times (1, \boldsymbol{i}_2)$
order 32	dimensionalization	$\times (1, \boldsymbol{j}_2)$
order 64	complexification	$\times (1, \boldsymbol{i}_3)$

The conjugation process only occurs once, because further applications would not change the character set, but the complexification and dimensionalization processes alternate to infinity. It is notable here that complex numbers are merely incomplete quaternion sets. It is significant, also, that order 16 is the point at which repetition begins.

4 The algebra applied to physics

Each of the processes involved in the generation of this algebra appears to have a realisation in physics, for we can easily identify the process of conservation with conjugation (meaning that the acquisition of a + value in a conserved or conjugated quantity can only happen if accompanied by the equivalent – value, and vice versa), and so write the table of parameters in the following form:

mass	conjugated	real	nondimensionalized
time	nonconjugated	complexified	nondimensionalized
charge	conjugated	complexified	dimensionalized
space	nonconjugated	real	dimensionalized

We can also see that, in addition to thus encoding the three processes involved in the algebraic structure on an equal basis, the parameters also incorporate the first four stages in the emergent algebra itself, and so reach the point of repetition:

order 2	real scalar	1	mass
order 4	pseudoscalar	i	time
order 8	quaternions	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	charge
order 16	multivariate vectors	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	space

Here, the multivariate vectors (or Pauli matrices) which appear to apply to space, with multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = 1$$
$$\mathbf{ij} = -\mathbf{ji} = i\mathbf{k}$$
$$\mathbf{jk} = -\mathbf{kj} = i\mathbf{i}$$
$$\mathbf{ki} = -\mathbf{ik} = i\mathbf{j}$$
$$\mathbf{ijk} = i \, .$$

are completely isomorphic to the complex quaternions which appear at this stage in the algebra:

$$(i\boldsymbol{i})^2 = (i\boldsymbol{j})^2 = (i\boldsymbol{k})^2 = 1$$
$$(i\boldsymbol{i})(i\boldsymbol{j}) = -(i\boldsymbol{j})(i\boldsymbol{i}) = i(i\boldsymbol{k})$$
$$(i\boldsymbol{j})(i\boldsymbol{k}) = -(i\boldsymbol{k})(i\boldsymbol{j}) = i(i\boldsymbol{i})$$
$$(i\boldsymbol{k})(i\boldsymbol{i}) = -(i\boldsymbol{i})(i\boldsymbol{k}) = i(i\boldsymbol{j})$$
$$(i\boldsymbol{i})(i\boldsymbol{j})(i\boldsymbol{k}) = i \, .$$

To incorporate the four algebras of space, time, mass and charge as independent units of a single comprehensive algebra, however, requires us to take our series to order 64, in the complex double quaternions or multivariate vector quaternions which appear in the Dirac equation, the fundamental equation needed to structure the whole of physics; and the Dirac equation, we will find, allows us a means of making an immediate return to zero at the same time as extending the algebra to infinity as required. This equation produces one of the most remarkable manifestations of 3-dimensionality in the whole of physics.

5 The Dirac state and 3-dimensionality

The conventional Dirac equation is structured on the 32-part algebra of the gamma matrices. Taking both + and – values, this forms a group of order 64 entirely isomorphic to the complex double quaternions or multivariate vector quaternions which emerge from the algebra discussed in section 3. A mapping between the two algebras can be established by forming composite terms within the multivariate vector quaternions, for example:

$$\gamma^o = -i\boldsymbol{i} \qquad \text{or} \qquad \gamma^o = i\boldsymbol{k}$$
$$\gamma^1 = \mathbf{i}\boldsymbol{k} \qquad\qquad \gamma^1 = \mathbf{i}\boldsymbol{i}$$
$$\gamma^2 = \mathbf{j}\boldsymbol{k} \qquad\qquad \gamma^2 = \mathbf{j}\boldsymbol{i}$$
$$\gamma^3 = \mathbf{k}\boldsymbol{k} \qquad\qquad \gamma^3 = \mathbf{k}\boldsymbol{i}$$
$$\gamma^5 = i\boldsymbol{j} \quad (1) \qquad\qquad \gamma^5 = i\boldsymbol{j}\ . \quad (2)$$

The binomial products of either of these 5-part sets or pentads will generate the entirely algebra in exactly the same way as the gamma matrices. When applied to the Dirac equation, set (1) can be converted to set (2) by multiplying the Dirac equation from the left by the equivalent of γ^5, to produce a nilpotent equation.

From the point of view of the fundamental algebra discussed in the previous sections, the pentad structures exemplified by (1) and (2) are the most efficient way of compactifying space, time, mass and charge into a single package. They are also a characteristic expression of the fundamental nature of 3-dimensionality and its relationship to closure. They produce a new infinite series of closed systems, which incorporate all the different processes and algebraic structures on an equal basis in such a way as to produce an immediate return to zero totality in each. Mathematically, the creation of a pentad involves taking the units of one of the two 3-dimensional components (the quaternion charge or vector space) and imposing each on the units of the other three parameters. We begin, for example, with:

time	space	mass	charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$

and impose each of the three charge units onto one of the algebraic expressions representing time, mass or space:

i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$
$\boldsymbol{k}$	$\boldsymbol{i}$	$\boldsymbol{j}$	

to give the following combinations:

$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}\ \ \boldsymbol{i}\mathbf{j}\ \ \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$

though, for mathematical convenience and for compatibility with the conventional way of writing the Dirac algebra, we will often write them in the form:

$$\boldsymbol{k} \qquad i\boldsymbol{i}\mathbf{i} \quad i\boldsymbol{i}\mathbf{j} \quad i\boldsymbol{i}\mathbf{k} \qquad i\boldsymbol{j} \quad .$$

We might expect the new, composite units to represent entirely new physical parameters, incorporating both the properties of charge, namely conservation and discrete quantization, and those of the respective parent parameters, time, space and mass. In fact, the very act of structuring the new quantities on a 3-dimensional (charge) substrate requires the resulting 3-dimensional combination to be discrete or quantized, though the new composite parameters, which we call the Dirac energy (*E*), the Dirac momentum (**p**) and the Dirac rest mass (*m*), will also retain the respective pseudoscalar, multivariate vector, and real scalar properties of time, space and mass:

$$\begin{array}{ccc} i\boldsymbol{k} & \boldsymbol{i}\mathbf{i} \;\; \boldsymbol{i}\mathbf{j} \;\; \boldsymbol{i}\mathbf{k} & \boldsymbol{j} \\ E & \mathbf{p} & m \end{array}$$

The concept of 'rest mass' only emerges in this process of quantization, and only exists in classical physics because it also exists in quantum physics, while the quantization of the directional properties of the vector term is expressed by relation to another quantized quantity, the Dirac angular momentum.

Dirac himself, on the basis of the quantization of angular momentum incorporated in the Dirac equation, apparently predicted that a magnetic monopole could exist with charge automatically quantized in integral multiples of fundamental constants, and that the existence of one such monopole anywhere in the universe would explain charge quantization. However, it would seem that it is rather the fundamentally quantized nature of charge that *explains* the quantization of angular momentum, and other quantities, in the Dirac state; so, the position is actually reversed.

As already stated, the use of a conserved quantity as substrate leads to *E*, **p** and *m* being conserved quantities in the Dirac state, but it is also possible to express the same superposition in the context of nonconservation, in terms of the quantum (or differential) operators, relating to the parent quantities, time and space:

$$\begin{array}{ccc} i\boldsymbol{k} & \boldsymbol{i}\mathbf{i} \;\; \boldsymbol{i}\mathbf{j} \;\; \boldsymbol{i}\mathbf{k} & \boldsymbol{j} \\ \partial/\partial t & \nabla & m \end{array}$$

although the object on which they act must be so structured as to produce the same conserved state as is represented by *E*, **p** and *m*. Exactly such a result is obtained by a differential operator acting on the exponential or 'wave' term, $e^{-i(Et\ -\ \mathbf{p.r.})}$, which can be seen as a mathematical representation of the group of space and time translations and rotations, which provide the maximal variation or 'nonconservation' for space and time coordinates in the most idealised or 'free' state. Because they lead to the same result, we can describe $\partial/\partial t$ as the operator E and ∇ as the operator **p**.

By necessity, quantization, in connecting all four parameters within a single dimensional structure necessarily establishes direct and inverse numerical relationships between their units. Through this process, E and t, and **p** and **r**, become conjugate variables, that is, ones which exchange statements about conservation into equivalent statements about nonconservation, and vice versa. The relationships between the units of E and **p**, and those of t and **r**, then lead to the introduction of the constants $\hbar$ and c, while a third constant, G, is required when we involve m. These constants, as has long been known, have no intrinsic meaning; they are simply the inevitable consequence of creating a composite state. With the explicit introduction of $\hbar$, the operator E becomes $i\hbar\partial/\partial t$, while the operator **p** becomes $-i\hbar\nabla$, though the usual convention is to choose units such that $\hbar = 1$ and $c = 1$.

Since the three components of the Dirac state, E, **p**, and m, are, from the fundamental properties of their parent-parameters time, space, and mass(-energy), specified by unrestricted real number values (though space's are countable in the Löwenheim-Skolem sense), it is possible, using the anticommuting properties of the quaternion and vector operators, and the presence of at least one complex term, to find values of the Dirac state, $(\pm \boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$, which square to a *zero numerical solution*. We may then use this property to define those states in which the conservation of E, **p**, and m, applies at the same time as the absolute nonconservation of space and time. The expression which results is the nilpotent Dirac equation, which in its purest form, for the free state, is given by:

$$\left(\pm i\boldsymbol{k}\frac{\partial}{\partial t} \pm \boldsymbol{i}\nabla + i\boldsymbol{j}m\right)\psi = \left(\pm i\boldsymbol{k}\frac{\partial}{\partial t} \pm \boldsymbol{i}\nabla + i\boldsymbol{j}m\right)(\pm \boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}\, m)\; e^{-i(Et - \mathbf{p.r})} = 0$$

The conjugate nature of E, **p** and t, **r** means that, through the Dirac equation, we can also establish a nilpotent structure connecting t and **r**, with another term τ (described as 'proper time') in the position occupied by m. The theory we know as 'special relativity' is merely the working out of the consequences of this structure, which we may write in the form $(\pm\, i\boldsymbol{k}t \pm \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau)$, under classical conditions. In fact, all other laws of physics may be seen in some sense as consequences of or approximations to the nilpotent Dirac equation.

Significantly, introducing the proper time term in the nilpotent expression ($\pm i\boldsymbol{k}t \pm \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$) also introduces the principle of *causality*, and this, along with relativity, is conventionally held to be the reason for the validity of the *CPT* theorem. The derivation of this theorem in section 7 depended on the necessity of the three terms in the nilpotent structure, ($\pm \boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m$) or ($\pm i\boldsymbol{k}t \pm \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$), having equal dimensional status, that is, on each having a quaternion operator. Causality, of course, is effectively a way of ensuring the irreversibility of time, and this, according to the symmetry group between the parameters presented in section 1, is equivalent to the unipolarity of mass, the term which occupies the 'proper time' slot in the energy-momentum nilpotent.

The nilpotency of the Dirac or fermion state, the fact that ($\pm \boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m$) squares to zero, gives us the opportunity of achieving the return to zero which was the original reason for the creation of the entire algebra, but we also need to extend the algebra to infinity. Here, we may consider the nilpotents ψ_1, ψ_2, ψ_3, …, with coefficients which are unrepeated but arbitrary units or strings of units of the form $\boldsymbol{i}_s$, as forming an infinite-dimensional Grassmann algebra, with successive outer products defined by the Slater determinant, and so requiring $\psi_1 \wedge \psi_1 = 0$ and $\psi_1 \wedge \psi_2 = -\psi_2 \wedge \psi_1$, etc. To create such an algebra, it would seem, the state vector units $\psi_{\boldsymbol{n}}$ must be both nilpotent and antisymmetric. The generating algebra which we have created from first principles can then be extended to infinity, through an algebraic and nonlocal superposition of fermionic states throughout the entire universe. Because we have an infinite range of real number values, we can consider each individual nilpotent to be unique, with superposition otherwise producing immediate zeroing – in a sense, an infinite number of individual or unique fermionic states, puts off the final return to zero, as each is examined against each other. It is this mathematical interconnectedness that allows us to group the nilpotents as closed 'units' of this even higher algebra, which is exactly equivalent to the conventional complex Hilbert space, and the nilpotency allows us to identify a part of the sequence without having to specify more than a finite number of the infinite number of terms which we know must exist. If the nilpotents were not themselves 3-dimensional, then this level of closure would not be possible.

6 Baryons

A classic example of the significance of 3-dimensionality occurs in the case of baryons. These structures only exist because the Dirac state vector incorporates the 3-dimensional term **p**. While it is clear that a state vector of the form

$$(\boldsymbol{k}E \pm \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$

would immediately zero itself, and so could not exist, this would not be the case with one of the form

$$(\boldsymbol{k}E \pm \boldsymbol{ii}\,p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,p_z + \boldsymbol{ij}\,m)\,,$$

where **p** may be imagined as having allowed phases in which only *one* of the three components of momentum, p_x, p_y, p_z, is nonzero and represents the total **p**. The products

$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$

would then become equivalent to the characteristic fermionic structure, $-p^2(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$, while

$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$

would result in $p^2(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$.

Assuming perfect gauge invariance, it is clear that these phases have exactly the same structure and $SU(3)$ symmetry as the conventional representation of the baryon, composed of three ‘coloured quarks’:

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB)\,,$$

with the mappings:

$$\begin{aligned}
BGR &\rightarrow (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\\
-BRG &\rightarrow (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\\
GRB &\rightarrow (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\\
-GBR &\rightarrow (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\\
RBG &\rightarrow (\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\\
-RGB &\rightarrow (\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,.
\end{aligned}$$

The behaviour of the strong interaction can then be understood in the most simple terms by mapping it onto the identical behaviour of the momentum or angular momentum operator **p**. We can even understand the perfect gauge invariance as a nonlocal 'transfer' of momentum between the phases at a rate independent of the separation of the component parts, which becomes equivalent to the linear potential used in the theory of the strong interaction. The principle may be expected to operate in relation to any states based on the 'quark' principle, or explicit use of the 3-dimensional properties of the **p** operator, including quark-antiquark as well as 3-quark states.

7 *CPT* symmetry

CPT symmetry is an even more obvious result of 3-dimensionality. The *P*, *T* and *C* transformations are equivalent to reversals in the signs of space, time and mass-energy, and can be accomplished by using the ***i***, ***k***, and ***j*** operators which connect these to the three dimensions of charge in the Dirac state vector:

Parity (*P*): $\boldsymbol{i}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{i} \;=\; (\pm\,\boldsymbol{k}E \mp i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)$

Time reversal (*T*): $\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{k} \;=\; (\mp\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)$

Charge conjugation (*C*): $-\boldsymbol{j}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{j} \;=\; (\mp\,\boldsymbol{k}E \mp i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)$

Obvious consequences of these are the combined transformations:

CP = *T*:
$-\boldsymbol{j}\,(\boldsymbol{i}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{i})\,\boldsymbol{j} = \boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k} = (\mp\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)$

PT = *C*:
$\boldsymbol{i}\,(\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k})\,\boldsymbol{i} \;= -\boldsymbol{j}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{j} = (\mp\,\boldsymbol{k}E \mp i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)$

TC = *P*:
$\boldsymbol{k}\,(-\boldsymbol{j}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{j})\,\boldsymbol{k} = \boldsymbol{i}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{i} = (\pm\,\boldsymbol{k}E \mp i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)$

and the fact that *TCP* ≡ identity, because:

$$\begin{aligned}\boldsymbol{k}\,(-\boldsymbol{j}\,(\boldsymbol{i}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{i})\,\boldsymbol{j})\,\boldsymbol{k} &= -\boldsymbol{kji}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{ijk} \\ &= (\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m).\end{aligned}$$

8 Symmetry breaking and 3-dimensionality

The combination of space, time, mass and charge in creating the Dirac state has another important physical consequence, as the quaternion units, ***i***, ***j***, ***k***, are changed from being symmetrical and indistinguishable representations of independent charges into composite units whose symmetry is broken, by being associated with quantities with different mathematical properties (pseudoscalar, vector and real scalar). From the composition of *i**k***, the combined (*i***i**, *i***j**, *i***k**), and ***j***, it is possible to derive the respective *SU*(2), *SU*(3) and *U*(1) symmetries associated with the weak, strong and electric charges.

*i**k***	*i***i** *i***j** *i***k**	***j***
w	*s*	*e*

But there is something even more fundamental at work here. *Any* 3-dimensional structure which has individually identifiable components is, in principle, a broken or chiral symmetry, and it is always broken in the same way. If we take, say, a quaternion system and identify ***j*** (the label is arbitrary, but this choice will be convenient), then we have, typically, a magnitude or a level of complexification. If, but only if, we bring in a second term, say ***i***, we will introduce dimensionalization, and it will necessarily be 3-dimensionalization, automatically generating ***k***. This will mean that the ***k*** term now has nothing left to do, except determine + or – values, or right- or left-handed axes. In a sense, ***k*** has been made redundant, except for ‘book-keeping’. Of course, where we don’t identify the axes, as for example in the usual description of space rotation, the perfect symmetry is preserved, and it appears that the symmetry-breaking has a close association with the use of a concept of conservation or conjugation in connection with the axes, the ‘book-keeping’ term being specifically concerned with this, and being of the opposite complexity to the rest to ensure the zeroing of the squared nilpotent quantity, while keeping open the two possible sign options.

The separate roles for the three axes in a 3-dimensional system with identifiable components has a remarkable similarity with the processes involved in creating the infinite algebra. The role of ***j*** is essentially that of complexification, the beginning of a new and as yet incomplete new quaternion system. The role of ***i*** is to introduce dimensionalization, while ***k*** is restricted to the ‘book-keeping’ role of conjugation or conservation. These also run parallel to the roles of scalar, vector and pseudoscalar quantities (which an extra *i* factor has transformed from the sequence pseudoscalar, quaternion, scalar). This is not, in fact, a coincidence, because the key properties of the fundamental parameters have been chosen, by a process of physical 'natural selection' of what can be made to 'work', to reflect the 3-dimensionality which makes it possible to define them at all. The same also applies to the parameter sequence mass, space, time, whose algebraic structures effectively reflect those attributable to the components of charge, the only fundamental 3-dimensional system with identifiable, i.e. independently conserved, components. It is this parallelism which makes it possible to create a closed parameter system with zero totality and in-built repetition.

A fundamental difference between charge and space, as 3-dimensional parameters, is that the first is a conserved quantity, whereas the second is not. One aspect of the nonconservation of space is its rotation symmetry, the principle that the laws of physics are invariant to the arbitrary rotation of spatial axes, a property which leads clearly to space's *affine* structure, the infinite number of possible resolutions of a vector into dimensional components. Clearly, this cannot apply to charge, whose conservation property must be exactly opposite. The axes of charge, that is, the electromagnetic, strong and weak types, must be rotation *a*symmetric: they cannot rotate into each other, and effectively constitute a non-affine 'space'. Charge type must be conserved. In fact, this principle is over and over again the message of particle physics. Despite the combined electroweak theory developed by Weinberg and Salam, and the proposed GUT unification of the electroweak with the strong force, the three nongravitational interactions each behave as if the others did not exist, and much of particle physics (lepton flavour conservation, baryon conservation, $SU(2)$ weak isospin, nondecay of the proton, etc.) is simply a statement of some aspect of this fact.

Now, the rotation symmetry of space, although an expression of nonconservation, is still responsible for a conservation law. This is a result of Noether's theorem, which states that, for every global transformation preserving the Lagrangian density, there exists a conserved quantity. This, however, is effectively a result of the exactness of the oppositeness of conservation and nonconservation in the parameter group. Noether's theorem has been taken, for instance, to imply that the translation symmetry of time is precisely identical to the conservation of energy, and that the translation symmetry of space is precisely identical to the conservation of linear momentum, while the additional rotation symmetry of 3-dimensional space becomes identical to the conservation of angular momentum. We can see how the conservation / nonconservation connection operates in the case of the first relation. Since energy is related to mass by the equation $E = mc^2$, then the translation symmetry of time will also be linked to the conservation of mass. So, the nonconservation of time is responsible for the conservation of mass, exactly as the parameter table would suggest.

We can, however, extend the interpretation of Noether's theorem even further by linking the conservation of the quantity of charge (of any type) with the nonconservation, or translation symmetry of space, and consequently with the conservation of linear momentum; and, by the same reasoning, we can make the conservation of *type* of charge linked to the rotation symmetry of space, and so to the conservation of angular momentum, as in the following scheme:

symmetry	**conserved quantity**	**linked conservation**
space translation	linear momentum	value of charge
time translation	energy	value of mass
space rotation	angular momentum	type of charge

Using the last connection, we can propose that, if conservation of angular momentum is also taken to represent conservation of type of charge, then any symmetry-breaking which differentiates types of charge will also be applicable to the quantized angular momentum relevant to particle physics. Remarkably, this appears to be the case, as the three charge types (electric, strong and weak) seem to be responsible for conveying different aspects of angular momentum conservation, as though these were representable by different identifiable dimensions of the quantity. In the electric case it is the magnitude; in the strong case the direction; and in the weak case the orientation. Again, we recognise the symmetry-breaking pattern which is characteristic of 3-dimensional systems with identifiable components: the magnitude, or complexifying, term; the dimensionalizing term; and the 'book-keeping' term providing the orientation. It has nothing to do with mysterious physical characteristics possessed by these interactions: it is a result of 3-dimensionality alone.

9 Fermionic structures

In section 6, we associated the *SU*(3) symmetry for the strong charge with the dimensional behaviour of the **p** operator. From the structure of the Dirac state vector, it is clear that this will be affected by the combination of the other two charges. However, these charges are actually governed by quite separate symmetries, the weak charge being attached to *iE* and the electric charge to *m*, in the Dirac state, and we can expect that the pseudoscalar nature of *iE* and the scalar nature of *m* will determine the respective characters of the weak and electric forces.

Now, the complex algebra determines that we have two sign options for *iE*, with two mathematical solutions, and consequently two helicity states; and it is, of course, the weak interaction that is concerned with this aspect of angular momentum conservation. However, the weak interaction has the special property of being confined to a single, left-handed, helicity state for fermions, with the right-handed state reserved for antifermions. This is entirely a result of the fundamental parameter group structure requiring mass-energy to be a continuum or non-dimensional quantity, and the consequent generation of a filled vacuum state; and it parallels the single physical (as opposed to mathematical) direction available to time.

In principle, there is no *physical* state corresponding to –*E*, although the use of a complex operator ensures that –*iE* has the same *mathematical* status as *iE*. Charge conjugation, however, or reversal of the signs of quaternion labels, *is* permitted physically, because charge is dimensional. So the –***ik****E* states can be interpreted as antifermion or charge-conjugated states; and the mass-energy continuum becomes a filled vacuum for the ground state of the universe, in which such states would not exist. The filled ***k*** or weak vacuum for the –*iE* fermion states, however, leads to a charge conjugation violation for the weak interaction, which manifests itself in the indifference of the interaction to the sign of weak charge, though not to the fermion / antifermion status. To preserve *CPT* symmetry, either parity or time-reversal symmetry must also be violated. In addition, when both *w* and *e* are present to affect **p**, the helicity state is no longer that of the pure *w*, and a mass term is generated, representing the scalar or magnitude part of the broken symmetry.

The $SU(2)_L$ or 'isospin' symmetry for the weak interaction now follows from the very principle which ensures that the 3-dimensional symmetry between the charges is a broken one – the fact that its component axes are separately identifiable because the three charges are conserved independently of each other. The mutual independence of weak and electric charges creates the $SU(2)_L$ weak isospin: the weak component acts in the same way, whether or not charges are present. The two $SU(2)_L$ states define the weak interaction, with and without electric charge. If we take the mixing of *E* and **p** terms, or right-handed and left-handed components, as being also equivalent to the mixing of *e* and *w* charges, this mixing will not affect the weak interaction as such. So, the weak interaction will be simultaneously left-handed for fermion states and indifferent to the presence or absence of the electric charge, which introduces the right-handed element.

The weak interaction must behave in such a way that the two possible isospin states are indistinguishable. Conventionally, these two states are described by ‘the third component of weak isospin’, t_3, by analogy with the *SU*(2) of spin, whose value is such that $(t_3)^2 = (½)^2$ in half the total number of possible states, that is, in the left-handed ones. The relevant quantum number for electric charge (*Q*) is determined by its absence or presence, and, for free fermions, takes the values 0 and –1, equivalent to the charges 0 and –*e*, the negative sign being purely historical in origin. So, once again, $Q^2 = 1$ in half the total number of possible states (though this time it is a different half, including the right-handed ones), and 0 in the others. Using the standard argument of Georgi and Glashow (1974), it can be shown that, if the weak and electric interactions are described by some grand unifying gauge group, irrespective of its particular structure, then orthogonality and normalisation conditions require the parameter describing the mixing ratio, $\sin^2\theta_W$, to be precisely determined by Tr $(t_3{}^2)$ / Tr (Q^2), which in this case must be 0.25.

The ratio cannot apply only to free fermions, as the weak interaction must also be indifferent to the presence or absence of the strong charge, or the directional state of the angular momentum operator. This means that the same mixing proportion must exist also for quark states, and separately for each ‘colour’ phase, so that colour is not directly detectable through *w*. Assuming that the same weak isospin states can be created for one lepton-like colour or phase, that is with alternative *Q* values of –1 and 0, or charge values of –*e* and 0, we now find that the only corresponding isospin states for the other colours that retain both the accepted value of $\sin^2\theta_W = e^2 / w^2$ in a system which allows the instantaneous existence of only one quark phase in three, are 1 and 0 (or *e* and 0). So, the charge variation 0 0 –*e* is taken against either an empty background or ‘electric vacuum’ (0 0 0) or a full background (*e e e*), so that the two states of weak isospin in the three colours become:

$$\begin{array}{ccc} e & e & 0 \\ 0 & 0 & -e \end{array} \; .$$

In this interpretation, the weak interaction has again performed its ‘book-keeping’ role, while the electromagnetic interaction takes on the required *U*(1) structure for a pure scalar magnitude by introducing a required phase. In more conventional terms, if *SU*(2) breaks parity, group structure and renormalizability require the incorporation of *U*(1). This also becomes significant in defining a Higgs ground state which is nonsymmetric and parity violating through identification of the one such state that *SU*(2) and *U*(1) have in common. The next section will show, however, that the origin of the phase term is evident in the solutions of the Dirac equations that preserve angular momentum conservation for single charges.

The symmetries, as defined in this section, effectively specify all possible fermion (that is, quark and lepton) states. Such states can be defined as all those *which are indistinguishable from each other in terms of the weak interaction*. Because of the 3-dimensional character of the strong interaction, quarks are not independent fermions, but merely phases of them. The phases are made explicit in the presence of the strong charge, in baryons and mesons, but are absent when the strong charge is absent, in free fermions or leptons. The weak and electric charges, unlike the strong charge, have no dimensional character, and only one phase, and so, where the strong charge is present, their phases cannot be aligned, as this would also confine the strong charge to a single phase. If the strong charge is absent, however, this alignment becomes necessary. This is the main distinction between quarks and leptons.

In terms of the weak interaction, however, quarks ought to be indistinguishable from leptons. The emergence of fractional *e* charges in QED phenomenology can therefore be taken as an expression of the perfect gauge invariance of the strong interaction. In this case, the 3-dimensional axes are not specifically identifiable and the symmetry remains unbroken. We may therefore propose that the charge structures for fundamental fermions are represented in the following tables, the left-handed quarks being represented by *A*, *B*, *C* and the leptons by *L*:

A

		B	*G*	*R*
u	+ *e*	1***j***	1***j***	0***i***
	+ *s*	1***i***	0***k***	0***j***
	+ *w*	1***k***	0***i***	0***k***
d	– *e*	0***j***	0***k***	1***j***
	+ *s*	1***i***	0***i***	0***k***
	+ *w*	1***k***	0***j***	0***i***

B

		B	*G*	*R*
u	+ *e*	1***j***	1***j***	0***k***
	+ *s*	0***i***	0***k***	1***i***
	+ *w*	1***k***	0***i***	0***j***
d	– *e*	0***i***	0***k***	1***j***
	+ *s*	0***j***	0***i***	1***i***
	+ *w*	1***k***	0***j***	0***k***

C

		B	*G*	*R*
u	+ *e*	1***j***	1***j***	0***k***
	+ *s*	0***i***	1***i***	0***j***
	+ *w*	1***k***	0***k***	0***i***
d	– *e*	0***j***	0***k***	1***j***
	+ *s*	0***i***	1***i***	0***k***
	+ *w*	1***k***	0***j***	0***i***

L

		$\bar{e}$	$\bar{e}$	ν_e
	+ *e*	1***j***	1***j***	0***j***
	+ *s*	0***k***	0***i***	0***i***
	+ *w*	0***i***	0***k***	1***k***
				e
	– *e*	0***i***	0***k***	1***j***
	+ *s*	0***j***	0***i***	0***i***
	+ *w*	0***k***	0***j***	1***k***

The filled nature of the weak vacuum and the consequent violation of charge-conjugation symmetry for the weak interaction, however, requires yet another application of the principle of 3-dimensionality to the tables. The fact that the weak interaction is indifferent to the sign of the weak charge, and responds (via the vacuum) only to the status of fermion or antifermion means that we must, additionally, define two further generations, replacing w by $-w$, and introducing respective violations of parity and time-reversal symmetry. The three quark-lepton generations are a consequence of the 3-dimensionality of the C, P and T symmetries.

It is possible to generate all the information incorporated in these tables using the angular momentum connection to provide a single unified representation for the entire set of charge structures for quarks and leptons (and their antistates) (Rowlands, 2003a):

$$\boldsymbol{\sigma}_z . (\boldsymbol{i}\, \hat{\mathbf{p}}_a\, (\delta_{bc} - 1) + \boldsymbol{j}\, (\hat{\mathbf{p}}_b - \mathbf{1}\delta_{0m}) + \boldsymbol{k}\, \hat{\mathbf{p}}_c\, (-1)^{\delta 1g}\, g) \quad .$$

The quaternion operators $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are respectively strong, electric and weak charge units; $\boldsymbol{\sigma}_z$ is the spin pseudovector component defined in the z direction (here used as a reference); $\hat{\mathbf{p}}_a$, $\hat{\mathbf{p}}_b$, $\hat{\mathbf{p}}_c$ are each units of quantized angular momentum, selected *randomly* and *independently* from the three orthogonal components $\hat{\mathbf{p}}_x$, $\hat{\mathbf{p}}_y$, $\hat{\mathbf{p}}_z$. These represent the phases of the respective direction, magnitude and orientation components of the angular momentum, determining the respective presence / absence of the units of strong, electric and weak charge. The other terms in the expression are merely codified ways of representing the divisions between fermions and antifermions, quarks and leptons, $SU(2)_L$ weak isospin, and the charge-conjugation violation associated with the weak interaction. The significant aspect of the expression, for our purposes, is the way it links the conservation of charge type and angular momentum through a broken 3-dimensional symmetry.

The quark tables may be taken as an illustration of the fact that broken 3-dimensional symmetries always incorporate unbroken ones. Thus, the tables are derived by assuming that an unbroken 3-dimensionality for colour phases lies within a broken one for charge, and can be derived, alternatively, by assuming that an irrotational 3-dimensional symmetry (charge conservation) is specified by a rotational one (quaternion algebra). (Interestingly, this alternative derivation *forces* the weak charge into adopting an ambiguous ± state.) It would seem, from fundamental considerations, that the broken 3-dimensionality represented by the Dirac state or charge conservation will necessarily include an unbroken one, such as the rotational symmetry of the **p** operator or the quark system. It is a characteristic, of course, of *nonconserved* or unbroken 3-dimensional structures that the dimensions themselves show the same structure, and this is the property responsible for the affine structure of space. In a sense it applies also to the non-vector terms in the broken symmetry. For example, m-**p**-E could be described as a 3-dimensional mass or energy and even as a 3-dimensional time (like τ-**r**-t), showing that 3-dimensionality is, in some sense, inherent within the whole parameter system described by the fundamental algebra. Unlike that of the vector term, however, these symmetries are not unbroken.

10 Spherically symmetric solutions of the nilpotent Dirac equation

According to the conception of Noether's theorem outlined in section 8, it ought to be possible to identify solutions of the nilpotent Dirac equation which involve spherically-symmetric distance-dependent potentials $V(r)$ as also being those which conserve angular momentum and therefore charge type. The procedure is relatively simple (Rowlands, 2003a). Using the standard conversion of the ∇ term to polar coordinates, with explicit introduction of fermionic spin (which is necessary only when we write ∇ as an ordinary vector), we set up an equation of the form:

$$\left(\boldsymbol{k}(E + V(r)) + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + ½}{r}\right) + \boldsymbol{ij}m\right) \psi = 0 \,,$$

where $V(r)$ is the r-dependent potential, and

$$\psi = \left(\boldsymbol{k}(E + V(r)) + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + ½}{r}\right) + \boldsymbol{ij}m\right) F(r, t)$$

and find the form of the phase term function $F(r, t)$, which will make ψ or its amplitude nilpotent. In a sense, we can avoid using the equation altogether and simply define the state vector, in differential form, as being a nilpotent. This will then uniquely determine both amplitude and phase in a way which is unique to the nilpotent method.

The simplest solution is found for the case where $V(r)$ is inverse linear ($\propto 1 / r$). This is characteristic of the electromagnetic or Coulomb interaction and emerges because of the inverse linear terms (due to spherical symmetry and spin) which are present in the $\boldsymbol{i}$ component of the state vector. A term of this kind in $V(r)$ is the minimum required for spherical symmetry and no such solution can be found without its presence. In effect, this potential gives the scalar part of the interaction, and it results in the characteristic scalar phase or $U(1)$ solution associated with the electromagnetic interaction. The phase term of the wavefunction becomes

$$F = e^{-ar}\, r^{\gamma} \sum_{\nu = 0} a_{\nu} r^{\nu} ,$$

and the available energy levels can be calculated from

$$\frac{E}{m} = \left(1 + \frac{(Ze^2)^2}{(\gamma + 1 + n')^2}\right)^{-1/2} = \left(1 + \frac{(Ze^2)^2}{(\sqrt{(j + ½)^2 - (Ze^2)^2} + n')^2}\right)^{-1/2} ,$$

which, for the case when $Z = 1$, becomes the standard 'hydrogen atom' solution.

If we now take $V(r)$ as a direct linear potential, combined with the inverse linear term which we know must be present, we obtain a solution with the characteristics of the strong interaction. The state vector has a functional component

$$F = \exp\,(\mp iEr \pm iq\sigma r^2/2)\; r^{\pm iqA - 1} \; .$$

in which the imaginary exponential terms can be seen as representing asymptotic freedom, the exp ($\mp iEr$) being typical for a free fermion. The complex $r^{\gamma-1}$ term can be written as a phase, $\phi\,(r) = \exp\,(\pm\, iqA \ln\,(r))$, which varies less rapidly with r than the rest of ψ. We can therefore write ψ as

$$\psi = \frac{\exp\,(kr + \phi\,(r))}{r} ,$$

where $k = (\mp\, iE \pm iq\sigma\, r/2)$. At high energies, where r is small, the first term dominates, approximating to a free fermion solution (asymptotic freedom). At low energies, when r is large, the second term dominates, with its confining potential σ (infrared slavery). The Coulomb or inverse linear term, which is required to maintain spherical symmetry, is, as we would expect, the component which here defines the strong interaction phase, $\phi\,(r)$, and this can be related to the directional status of **p** in the state vector. The direct linear term can be seen as equivalent to a force or rate of change of momentum which is constant with separation, and hence to a quantity which is determined by the vector nature of **p**.

The solutions for direct linear and inverse linear potentials, however, appear to be special cases, and they correspond exactly to the special cases found in classical physics, where they are held to be characteristic of 3-dimensional systems in steady state. In the case of the nilpotent Dirac equation, there appears to be only one other spherically symmetric solution, and it appears to be the same for any potential depending on r^n, where $2 \geq n \geq -2$. Any such potential, or one containing any combination of such terms, gives a harmonic oscillator solution, with energy levels

$$E = -\frac{m}{(j + ½)}(½ + n') \ ,$$

but only when combined with an inverse linear or Coulomb term of the opposite complexity, and the form of the solution is indifferent to the particular value of *n* chosen.

The harmonic oscillator, of course, can be expressed in terms of the creation and annihilation operators which characterize the unique behaviour of the weak interaction in creating and annihilating fermion-antifermion states, and it is entirely within our expectations of the principle of 3-dimensionality that one interaction should have this conjugative or ‘book-keeping’ role, after the others have dealt with the scalar magnitude and vector aspects. It corresponds to the position of the energy term in the Dirac operator which only has meaning in connection with fermion / antifermion, right- or left-handed, + or –; and relates to the fundamental process of conservation or conjugation. (This is why the weak interaction responds only to the status of fermion / antifermion and not to the sign of weak charge.) And, of course, values of *n* different from 1 or –1 will be expected from an interaction which is *invariably dipolar*, as the weak interaction certainly is. The dipolarity is a characteristic expected of a state determined by a pseudoscalar or imaginary quantity, with a ± mathematical duality, and this pseudoscalar aspect appears to be reinforced by the relative complexity of the r^n and inverse linear potential terms required for this solution. It is the same dipolarity as is found in the energy terms in the Dirac equation and in the time term in time-reversal symmetry.

It would seem from our analysis that if we take the Coulomb terms relating to all three interactions to be an expression of the real scalar magnitudes of the charges with which they are associated, then we may suppose that the additional potentials required by the ‘strong’ and ‘weak’ solutions are expressions of the respective vector and pseudoscalar terms associated, in the Dirac equation, with their charges. It would also seem that the Dirac equation produces three spherically symmetric, i.e. 3-dimensional, solutions because of its fundamental structural basis in a 3-dimensional object with individually identifiable components.

11 ‘4-dimensional’ space-time

Minkowski famously said about space and time, after his introduction of 4-vectors (1909): ‘From now on, space by itself, and time by itself, are destined to sink into shadows, and only a kind of union of both to retain an independent existence’. Of course, space and time still *are* connected, but the connection is not privileged as Minkowski believed it to be. The connection between space and time is no more significant than that between space and mass and mass and time, or all these parameters and charge. And *there is no fundamental 4-dimensional link between space and time.* There is, however, a 3-dimensional one!

The space-time 4-vector has always run into the problem that one component is physically different from all the others, and it is essentially on account of these differences that the problem of wave-particle duality developed. Wave theories made space timelike (i.e. continuous) while particle theories made time spacelike (i.e. discrete) to fit the two parameters into a single physical model or dimensional structure. The dichotomy even manifested itself in nonrelativistic quantum mechanics, with Schrödinger’s timelike theory opposed to Heisenberg’s spacelike one. However, the problem, in fundamental terms, is that it cannot be done. The true picture is restored in the Dirac nilpotent theory which is neither wavelike nor timelike, but incorporates elements from both Schrödinger and Heisenberg.

What this theory tells us is that space and time are not a 4-vector. We do not add a pseudoscalar to a pure vector, because each term is premultiplied by a gamma factor or a quaternion before addition. Space and time are actually two dimensions of a 3-dimensional structure, whose third dimension is a mass-related term, the ‘proper time’, which is of course premultiplied by the remaining quaternion. The ‘proper time’ is not a time term; it is not a pseudoscalar. It gets its name simply from the fact that it becomes *numerically* equal to the time variable if we equate the space component to zero. We could just as easily describe the actual time variable as the ‘proper space’ in investigating systems, such as photons, in which the proper time (or rest mass) becomes zero.

Of course, when we take a scalar product, as we invariably do in classical special relativity, the quaternion terms disappear and time acts, to all intents and purposes, as an imaginary fourth dimension of space, fulfilling the role of pseudoscalar needed to complete a mathematical vector theory. However, it is important that it is not exactly that pseudoscalar, and no physical quantity exists which can fill this role. The algebraic structure which we have created as a representation of physical ‘reality’ has no place for 4-dimensional physical quantities. It forces us over and over again into a 3-dimensional pattern. Our quantized, i.e. 3-

dimensional, picture denies us the opportunity of representing time as a fourth dimension, denying it status as a physical observable. In a *3-dimensional* theory, time occupies the place of the 'book-keeper', as energy does in the Dirac state, the quantity which preserves conservation or conjugation, but adds only the information of + or –. We only know the direction of the sequence that preserves causality, not a *measure* of time in the same sense as we measure space, in the same way as energy only tells us whether the system is a fermion or antifermion. This fact is well known as a stumbling block to proponents of a quantum theory of gravity, which automatically incorporates time as a physical fourth dimension. It is likely to prove equally damaging to string theories in which a spatio-temporal dimensionality is automatically assumed to be possible.

The fact that the number system we use in mathematics has a 3-dimensional origin is of profound significance. It means that we can't arbitrarily choose the number of dimensions we apply to quantities like space and time without contradicting the principles on which these concepts, and related ones, such as quantization and conservation, were founded. The number of dimensions is not negotiable once we have decided to use the fundamental parameter group and the number system which emerges from 3-dimensionality. Only at the level of classical approximation can we even contemplate any interference in the number of dimensions which nature appears to have thrust upon us.

Appendix I: Table of 3-dimensional systems with identifiable components

pseudoscalar	quaternion	scalar	(1)
scalar	vector	pseudoscalar	(2)
mass	space	time	
m	**p**	E	
τ	**r**	t	
e	s	w	
C	P	T	
j	***i***	***k***	
magnitude	direction	orientation	
complexification	dimensionalization	conjugation	
complexification	dimensionalization	conservation	

Here, the 'dimensional' term is in the second column and the 'book-keeping' term in the third. (1) = (2) × *i* and (2) = (1) × *i*. It may be that we can also include momentum-angular momentum-energy and space translation-space rotation-time translation. The last row refers to the properties of the parameter group, whose fundamental 3-dimensionality is displayed in the diagrams included in Rowlands (2003b).

Appendix II: Quantum gravitational inertia

The principle that 3-dimensionality is the sole source for discreteness in physics, and that no other dimensionality exists at the fundamental level has consequences for the development of a mathematical theory of quantum gravity, or, in more accurate terms, a mathematical theory of quantum gravitational inertia. According to the argument presented here, there is no fundamental 4-D, and, though there is a mathematical object called a 4-vector, there is no physical realisation of it, except in the classical approximation. The key structure then becomes the *3-dimensional* nilpotent structure, variously represented by $\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$ and $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$, which is both fully quantum and fully relativistic, and the 3-dimensionality of the structure is essential to its complete quantization.

There is no true 5-dimensionality in the structure, as we might at first think, because the nonconserved 3-dimensionality of the **p** and **r** terms is of a different nature to the conserved 3-dimensionality of $\boldsymbol{k}$, $\boldsymbol{i}$ and $\boldsymbol{j}$, though we can, if we choose, relate the nilpotent information in $\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$ and $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$, as defining the ten ‘degrees of freedom’ concerning any fermionic state which lie at the basis of the 10- and 11-dimensional superstring and supermembrane theories. The possibility of establishing such a connection, and the outline method of achieving it, are discussed in Rowlands (1998) and Rowlands *et al* (2001). The Grassmann algebra linking the nilpotent fermionic states (which is equivalent to the conventional Hilbert space) would provide the so-called ‘eleventh dimension’. However, although it is worth showing that this is possible, it is not worth pursuing in detail, as there is no point in developing a more limited superstructure, whose ultimate purpose is to provide a route to a more fundamental basis, when that basis is already available. Thus, although various larger algebraic structures, for example octonions and even classical Minkowski space-time, have been shown to produce some of the results that are required in a fundamental theory, this is always at the price of producing others which are invalid, and it would seem that the 3-dimensional pattern is the one that nature prefers, and that in identifying this as the true fundamental context we are likely to discover more universally valid results.

We can now, for example, immediately relate $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$ to the discrete gravity theory presented in Koberlein (2001), which is based on the fact that a single object (particle or field) at two points in Minkowski space-time (represented by the 4-vector x) must satisfy the causality constraint $\Delta\tau^2 + \Delta x^2 = \Delta(\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau)^2 = 0$, which defines a hypercone for the object. 'Extended causality' then applies when we shift τ and x by infinitesimal steps $d\tau$ and dx. Using Koberlein's procedure, we can then apply a massless scalar field to obtain a discrete field equation, and a field source represented by a scalar charge to generate a 'graviton'-like object and a metric for a discrete gravitational field. It is clear that if we apply the quantum $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$ for a Dirac particle in the appropriate places in place of (x, τ), then we can produce a fully quantum version of this discrete gravity, with the discreteness referring to an interaction between fermions as discrete particles defined by a 3-D Dirac nilpotent. Significantly, the discrete theory also dispenses with the transverse directions, to create a 1 + 1 space-time, paralleling the fact that a quantum Dirac particle, with conserved charge (the kind of object to which quantum gravity or quantum gravitational inertia will apply), requires only $\mathbf{r}$, and a single well-defined direction of spin, rather than the classical x, y, z.

It is already apparent, from previous quantum gravity theories, that any attempt at quantizing 4-D space-time is a lost cause, because time is not an observable in quantum mechanics as it is in classical relativity theory; it merely plays the 'book-keeper' role of specifying the direction which preserves causality. This means that, for a fully quantized theory, we need a metric other than the 4 × 4 representation using x, y, z, t, with added curvature, which is used in classical general relativity. The obvious one that suggests itself is a 3 × 3 representation, with diagonal terms $\boldsymbol{ik}t$, $\boldsymbol{i}\mathbf{r}$, $\boldsymbol{j}\tau$, in the absence of the curvature resulting from a gravitational field. This formalism would have the distinct advantage of being a natural 2 + 1 theory of gravity (the 2 representing the 'real' terms $\mathbf{r}$ and τ, and the 1 the imaginary term it), and such theories are already known to be renormalizable, unlike those with a higher number of dimensions. A preliminary investigation of the method suggests that it works exactly as expected.

Now, Bell *et al* have presented a preliminary approach to a QED-like quantum gravity (2002) by using a quaternionic mapping of the four solutions of the Dirac equation onto a space which, without curvature, is equivalent to that provided by the usual 4 × 4 representation of the Lorentzian metric. The natural result of this mapping is the production of the Bohr-Sommerfeld orbitals for the electron in a scalar electrostatic potential in a purely classical way, thus providing a 'natural' generation of space-time curvature, which can be extended when gravitational curvature terms are directly applied to the metric of the four Dirac solutions (Bell *et al*, 2000).

In terms of the theory presented here, of course, any version of the 4 × 4 Lorentzian metric will be neither fully quantized nor fully relativistic, but the 3 × 3 ‘quantum metric’, based on ***i**kt*, ***i*r**, ***j**τ*, will fulfil both these criteria, and, in the spirit of Bell *et al* (though using a different set of dimensional quantities), can be mapped onto a ‘phase space’ metric based on ***i**kE*, ***i*p**, ***j**m*, which gives the full information about the Dirac state, and produces the full Dirac ‘atom’ solution and *U*(1) QED-type behaviour, with a corresponding photon-like mediating boson, merely on the assumption of spherical symmetry and the multivariate vector nature of the spin term **p** (or, equivalently, conserved charge) (Rowlands, 1992, 1994). A purely ‘Lorentzian’ metric would not, of course, automatically include spin, unless the vector term was assumed to be multivariate, but, more seriously, would exclude the fundamental nilpotent relations between the parameters space, time, mass and charge which are responsible for both quantization and relativity.

The phase space metric has the direct advantage that it can be obtained directly from the ‘quantum metric’ (and vice versa) via a Fourier transform, and we can thus imagine the quantum metric as being generated by and carried along with the state which defines it. Evidence for ‘curvature’ (i.e. the effect of a gravitational field on the inertial metric) can then be seen in the functional term through which this transformation occurs – which will be the usual complex exponential for a free particle, but distorted in the presence of a field or ‘curvature’. Since the Dirac state directly determines the nature of the vacuum which responds to it, this process will be equivalent to the Davies-Unruh effect, where a nonaccelerating system sees a plane-wave version of the zero-point field but an accelerating system sees a distorted one.

In the case of phase space, the reduction of the metric to 3 × 3 reflects the fact that, in the nilpotent formulation, the specification of four separate solutions becomes redundant information in the Dirac spinor, because knowledge of the signs of ***i**kE* and ***i*p** in the first term automatically gives us the entire pattern which follows – this is equivalent to separate specification of *x*, *y* and *z* being redundant in the quantum context. In addition, if the basic metric is 3 × 3, rather than 4 × 4, the mediator responsible for any curvature terms becomes spin 1 (as Bell *et al* require for a renormalizable theory), rather than spin 2.

The need for a spin 1 mediator and QED-like theory in ‘quantum gravity’ has been discussed in many previous publications. There, it has been suggested that the continuity of mass-energy, the filled vacuum, the Higgs field, and the need for instantaneous correlation between Dirac states, together with the fact that energy does not actually move (as opposed to the form of its realisation in connection with a discrete state), require an instantaneous gravitational force, which is undetectable by direct observation, and only ever observed through the *c*-dependent inertial reaction on discrete fermionic or bosonic states. Being repulsive, this force requires a mediator of spin 1. In this context, we may note that the nilpotent representation significantly makes the Dirac state identical to its own gravitational vacuum, at 1(***ik****E* + ***i*****p** + ***j****m*), whereas the vacuum responses to the weak, strong and electric charges can be represented, respectively, by ***k***(***ik****E* + ***i*****p** + ***j****m*), ***i***(***ik****E* + ***i*****p** + ***j****m*), and ***j***(***ik****E* + ***i*****p** + ***j****m*).

The standard mathematical representation of the gravitational force incorporates no information relating to speed, but the description of gravity as an undetectable property of the vacuum would *require* it to be instantaneous. The *c*-dependence of the inertial reaction, however, determines that, though linear and renormalizable, this force will itself be affected by gravity, giving rise to the 'curvature' terms in the metric tensor, as in general relativity. It is, however, 'curvature' of the metric for inertia, not for gravity, which has no metric. Previous work has shown that, if we equate the inertial reaction numerically with the undetectable gravitational attraction (so defining an equivalence principle), we justify a form of Mach's principle, and obtain gravomagnetic effects, redshift, acceleration of the redshift, and perhaps even the cosmic microwave background radiation (Rowlands, 1992, 1994, 2002). In the simplest case of 'curvature', provided by a point source, we will generate the Schwarzschild metric and a factor 4 in the gravomagnetic equations by comparison with those for QED. This factor (incorporating 2 for space 'contraction' and 2 for time 'dilation', if we adopt the usual convention of making *c* constant, is evident in the factors of 2 which appear in the mass and field terms in the Schwarzschild solution presented by Bell *et al* (2002).

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