A Hierarchy of Symmetries

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Abstract. Symmetry occurs at all levels in Nature and provides one of the prime methods of scientific investigation, reflecting its role in our evolution as a species. At the most fundamental level physics appears to be governed by a hierarchy of symmetries, beginning with the Klein-4 group structure connecting the fundamental parameters mass, time, charge and space. The algebras associated with these parameters emerge can be shown to emerge in a sequence which successively generates real numbers, complex numbers, quaternions and multivariate vectors. Remarkably, the combined algebra appears to be identical to that of the Dirac equation of relativistic quantum mechanics, the equation that applies to the point-like fermion, the most fundamental physical state. Other significant symmetries and the symmetry-breaking mechanism between the four physical interactions can be shown to emerge from this foundational symmetric structure.

Introduction

Although humans are not very good observers and find science a struggle, one particular talent developed during our evolution continues to serve us well. This is pattern recognition. The fact that we have evolved to recognise pattern and that pattern, in the form of symmetry, is found in many places at the deepest and most foundational level in physics, suggests that it is a recurring, possibly even fractal, organizing principle in Nature.

Significantly, the symmetries observed in fundamental physics often appear to be ‘broken’, that is disguised or hidden. Space and time provide a classic, but little recognised, case for though they can be combined in a higher, ‘4-dimensional’, structure in special relativity, the two parameters maintain many properties which establish them as fundamentally different.

Clearly, symmetry-breaking is an important phenomenon and provides a level of structure which may allow new insights to emerge, but we have to establish why some symmetries broken and what broken symmetry really means. In addition we need to identify which symmetries are the most important, and where symmetry actually comes from, and also how the most fundamental symmetries could enable us to construct physics as we know it. In addition, many symmetries are expressed in some way using integers, and we have to establish which are the most important?

The most successful method would be to allow one symmetry to comment on another. Here, I would like to propose that there is a hierarchy of symmetries, emerging at a very fundamental level, all of which are interlinked. We can also propose a philosophical starting-point in that the ultimate origin of symmetry in physics is zero totality, and that the sum of every single thing in the universe is precisely nothing. Nature as a whole has no single definable or universal characteristic. Making zero the only logical starting-point, and the only idea we can’t conceivably explain, means that we have a starting point that we don’t need to explain. Now, it is possible to develop this idea in a fundamental way, but, here, we will use semi-empirical reasoning. The major symmetries in physics can be shown to begin with just two ideas:

duality and anticommutativity

Connected with these are just two fundamental numbers or integers:

2 and 3

Everything else is just a variation of these concepts and numbers. In a more physical perspective, anticommutativity is like creation, duality is like conservation. In information terms, they are aspects of a universal rewrite system, developed from an infinite succession of zero totalities (Rowlands and Diaz, 2002, Rowlands, 2007, 2014, 2010b).

Space, time, mass and charge

Let us start with a symmetry that is not well known, but which I have long proposed as foundational to physics (Rowlands, 1979, 1982, 1982, 1991, 2001, 2002, 2007, 2009, 2014, 2015). This is the one between the four fundamental parameters

SPACE TIME MASS CHARGE

Mass here incorporates energy as well as invariant or rest mass, and charge includes the sources of all 3 gauge interactions (electric, strong and weak). The charges exhibit a well-known broken symmetry, but we will assume that this is an emergent property, and we will show later that it emerges from algebra.

The properties of these parameters can be arranged symmetrically as a Klein-4 (D_2) group:

space	nonconserved	real	anticommutative
time	nonconserved	complex	commutative
mass	conserved	real	commutative
charge	conserved	complex	anticommutative

A very large number of physical, and also some mathematical, facts, which have not been fully understood, emerge directly as simple consequences of this symmetry:

The conservation laws
Noether's theorem
The irreversibility of time
The unipolarity or single sign of mass
The repulsion of like charges repel but attraction of like masses
The need for antistates, even to electrically neutral particles
Lepton and baryon conservation
The nondecay of the proton

Standard and nonstandard analysis, arithmetic and geometry
Zeno's paradox
The irreversibility paradox
Gauge invariance
Translation and rotation symmetry

Representations of the parameter group

As many of years of testing have shown, the symmetry within the parameter group is absolutely exact, and probably the most exact in the whole of physics. A key aspect of this exactness is that space, if it is truly symmetrical to charge in its anticommutativity or 3-dimensionality, is not just an ordinary vector, but one which has the structure of a Clifford algebra:

i **j** **k**	vector		
*i***i** *i***j** *i***k**	bivector	pseudovector	quaternion
i	trivector	pseudoscalar	complex
1	scalar		

The space-time and charge-mass combinations then become exact mirror images, 3 real + 1 imaginary against 3 imaginary + 1 real. The vectors of space and physical quantities derived from it are what Hestenes (1966) called multivariate vectors. They are isomorphic to both Pauli matrices and complexified quaternions, with a full product

$$\mathbf{ab} = \mathbf{a.b} + i\, \mathbf{a} \times \mathbf{b}$$

which comes with a built-in concept of spin, deriving from the $i\, \mathbf{a} \times \mathbf{b}$ term. One of Hestenes' most significant results demonstrated that if we use the full product $\nabla\nabla\psi$ for a multivariate vector ∇ instead of the scalar product $\nabla.\nabla\psi$ for an ordinary vector ∇, we can obtain spin ½ for an electron in a magnetic field from the nonrelativistic *Schrödinger equation* without introducing any quantity of relativity.

With exact symmetry assumed between the parameters, space and time become a 4-vector with three real parts and one imaginary, by symmetry with the mass and charge quaternion, with three imaginary parts and one real.

space	time	charge	mass
$\mathbf{i}x\ \mathbf{j}y\ \mathbf{k}z$	it	$\boldsymbol{i}s\ \boldsymbol{j}e\ \boldsymbol{k}w$	$1m$

The vector units, like those of the quaternions, are also anticommutative.

A very simple representation of the group properties uses algebraic symbols for the properties / antiproperties:

mass	*x*	*y*	*z*
time	–*x*	–*y*	*z*
charge	*x*	–*y*	–*z*
space	–*x*	*y*	–*z*

In algebraic terms, this is a conceptual zero.

There is also a dual version of this group, which reverses one set of properties / antiproperties, say the first:

mass*	–*x*	*y*	*z*
time*	*x*	–*y*	*z*
charge*	–*x*	–*y*	–*z*
space*	*x*	*y*	–*z*

The physical meaning of this will become clear later.

The C_2 symmetry between the dual D_2 structures allows us to create larger structure $C_2 \times D_2$ of order 8 with the form:

*	*M*	*C*	*S*	*T*	*M**	*C**	*S**	*T**
M	*M*	*C*	*S*	*T*	*M**	*C**	*S**	*T**
C	*C*	*M**	*T*	*S**	*C**	*M*	*T**	*S*
S	*S*	*T**	*M**	*C*	*S**	*T*	*M*	*C**
T	*T*	*S*	*C**	*M**	*T**	*S**	*C*	*M*
*M**	*M**	*C**	*S**	*T**	*M*	*C*	*S*	*T*
*C**	*C**	*M*	*T**	*S*	*C*	*M**	*T*	*S**
*S**	*S**	*T*	*M*	*C**	*S*	*T**	*M**	*C*
*T**	*T**	*S**	*C*	*M*	*T*	*S*	*C**	*M**

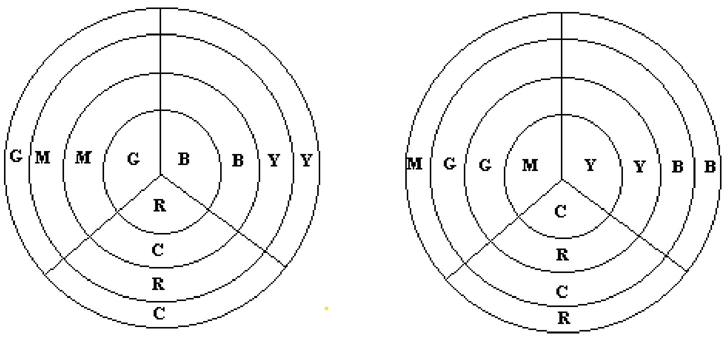
This structure is identical to that of the quaternion group (Q_8):

*	1	***i***	***j***	***k***	–1	–***i***	–***j***	–***k***
1	1	***i***	***j***	***k***	–1	–***i***	–***j***	–***k***
i	***i***	–1	***k***	–***j***	–***i***	1	–***k***	***j***
j	***j***	–***k***	–1	***i***	–***j***	***k***	1	–***i***
k	***k***	***j***	–***i***	–1	–***k***	–***j***	***i***	1
–1	–1	–***i***	–***j***	–***k***	1	***i***	***j***	***k***
–***i***	–***i***	1	–***k***	***j***	***i***	–1	***k***	–***j***
–***j***	–***j***	***k***	1	–***i***	***j***	–***k***	–1	***i***
–***k***	–***k***	–***j***	***i***	1	***k***	***j***	–***i***	–1

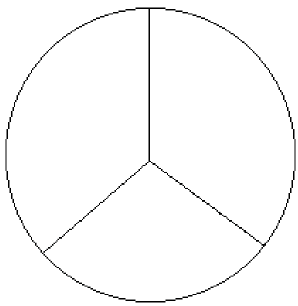
3-dimensionality is clearly at the very heart of this approach to physical symmetry.

The absolute exactness of the symmetry is manifested by the fact that no exception to the rule has been found in forty years. The condition can, therefore, be used to put constraints on physics to derive laws and states of matter, while a number of visual representations not only show the absoluteness of the symmetry, but also the centrality of the idea of 3-dimensionality to the whole concept. Because we have a perfect symmetry between the 4 parameters we need assume the properties of only one of them, with the others emerging automatically like kaleidoscopic images. As the visual representations further show, it is also completely arbitrary which parameter we assume to begin with.

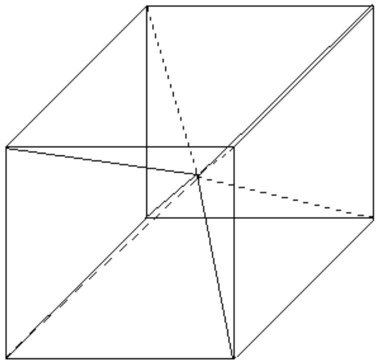
In the colour representation, we use primary colours (R, B, G), in any order, to represent the properties and secondary colours (C, Y, M) to represent the corresponding antiproperties, or vice versa. These are arranged in three sectors within concentric circles, each of which represents a parameter. The two diagrams shown could be a simultaneous representation of the group and the dual group in either order.



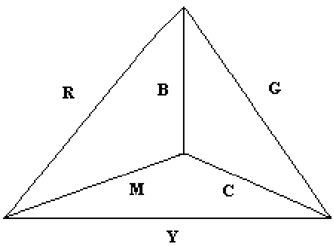
The coloured sectors sum to zero over all the parameters, indicating totality zero.



The 3-D representation is essentially a direct Cartesian plot of the properties as *x*, *y*, *z* and the antiproperties as –*x*, –*y*, –*z*, from an origin at the centre of a cube to four of its corners. The dotted lines are a representation of the dual group.



The tetrahedral representation has the same in-built duality as the other two. This time the parameters are represented at the four faces or vertices, with the edges taking on the primary / secondary colours associated with the six properties / antiproperties.



Algebra and the parameters

Significantly, the parameters and their properties are *purely abstract*, reducible, in effect, to pure algebra. This is obvious in the case of the Real / Imaginary and Commutative / Anticommutative divisions, but that between the Conserved / Nonconserved properties can also be shown to be algebraic in origin in the same way. The parameters are, in effect, defined by their own unique algebras, emerging from their unique combinations of properties and antiproperties. Their 'physical' characteristic come solely from these algebras.

Mass	1	scalar
Time	*i*	pseudoscalar
Charge	***i j k***	quaternion
Space	**i j k**	vector

The algebras of Mass, Time and Charge are subalgebras of the algebra of Space, and combine to produce an algebraic equivalent, let's say **I J K**. They are, in effect, equivalent to a 'vector space', an 'antispace' which counters **i j k**, the algebra of real space (Rowlands, 2013). It is clear from this why space appears to have a privileged status as the only parameter of direct measurement.

Now, one of the key aspects of the exactness of the symmetry between the parameters is that space, to be truly symmetrical to charge in its 3-dimensionality, is not just an ordinary vector, but one which has the properties of a Clifford algebra:

i **j** **k**	vector		
*i***i** *i***j** *i***k**	bivector	pseudovector	quaternion
i	trivector	pseudoscalar	complex
1	scalar		

It has 3 subalgebras:

bivector / pseudovector / quaternion, composed of:

*i***i** *i***j** *i***k**	bivector	pseudovector	quaternion
1	scalar		

trivector / pseudoscalar / complex, composed of:

i	trivector	pseudoscalar	complex
1	scalar		

and scalar, with just a single unit:

1	scalar

Reversing this decomposition, the three parameters other than space produce a combined vector-like structure, even though there is no physical vector quantity associated with them.

mass	scalar		1	
time	pseudoscalar	*i*	1	
charge	quaternion	***i j k***	1	
	pseudovector	*i***i** *i***j** *i***k**	1	
	bivector			
COMBINED	vector	**i j k**	***i j k*** *i*	1
STRUCTURE		**i j k**	*i***i** *i***j** *i***k** *i*	1

This is what we will call vacuum space.

In addition, with the algebras of charge, time, mass identified as subalgebras of vector algebra, it appears that, though all the parameters are equivalent in the group structure, they also produce a mathematical hierarchy, suggesting an ‘evolutionary’ structure in a logical, rather than a time sequence. This evolution has been derived from a universal rewrite system, and applied much more generally as a fundamental information process, with operations in mathematics, computer science, chemistry and biology, as well as in more complex aspects of physics.

Many aspects of the complexity may also be derived more directly, by investigating the ‘packaging’ which results from combining the separate sources of information obtained from the individual algebras. If we take our basic units of information in the form

Time	Space	Mass	Charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$
pseudoscalar	vector	scalar	quaternion

we can work out every possible combination of them in the form of 64 algebraic units. The algebra of this combination turns out to be that Dirac equation, the relativistic quantum mechanical equation of the fermion, which is the only true fundamental object that we know must exist.

The combinations can be broken down as

$(\pm 1, \pm i)$	4	units
$(\pm 1, \pm i) \times (\mathbf{i}, \mathbf{j}, \mathbf{k})$	12	units
$(\pm 1, \pm i) \times (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	12	units
$(\pm 1, \pm i) \times (\mathbf{i}, \mathbf{j}, \mathbf{k}) \times (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	36	units

which gives us + and – versions of the units:

$\boldsymbol{i}$	$\boldsymbol{j}^*$	$\boldsymbol{k}$	$i\boldsymbol{i}$	$i\boldsymbol{j}$	$i\boldsymbol{k}^*$	i	1
$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$	$i\mathbf{i}$	$i\mathbf{i}$	$i\mathbf{k}$		
$\mathbf{i}\boldsymbol{i}^*$	$\mathbf{i}\boldsymbol{j}$	$\mathbf{i}\boldsymbol{k}$	$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$		
$\mathbf{j}\boldsymbol{i}^*$	$\mathbf{j}\boldsymbol{j}$	$\mathbf{j}\boldsymbol{k}$	$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$		
$\mathbf{k}\boldsymbol{i}^*$	$\mathbf{k}\boldsymbol{j}$	$\mathbf{k}\boldsymbol{k}$	$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{i}$		

These units form a group of order 64. The simplest starting point for a group is to find the *generators*, the set of elements within the group that are sufficient to generate it by multiplication. There are many such sets; one such set is identified here, marked *.

Now, since vectors are complexified quaternions and quaternions are complexified vectors, we obtain an identical algebra if we use complexified double quaternions:

$\boldsymbol{i}$	$\boldsymbol{j}^*$	$\boldsymbol{k}$	$i\boldsymbol{i}$	$i\boldsymbol{j}$	$i\boldsymbol{k}^*$	i	1
$\boldsymbol{I}$	$\boldsymbol{J}$	$\boldsymbol{K}$	$i\boldsymbol{I}$	$i\boldsymbol{J}$	$i\boldsymbol{K}$		
$\boldsymbol{iI}^*$	$\boldsymbol{iJ}$	$\boldsymbol{iK}$	$i\boldsymbol{iI}$	$i\boldsymbol{iJ}$	$i\boldsymbol{iK}$		
$\boldsymbol{jI}^*$	$\boldsymbol{jJ}$	$\boldsymbol{jK}$	$i\boldsymbol{jI}$	$i\boldsymbol{jJ}$	$i\boldsymbol{jK}$		
$\boldsymbol{kI}^*$	$\boldsymbol{kJ}$	$\boldsymbol{kK}$	$i\boldsymbol{kI}$	$i\boldsymbol{kJ}$	$i\boldsymbol{kK}$		

A third may be derived using double vectors. This emphasizes the algebra's origin in a double space.

$\mathbf{i}$	$\mathbf{j}$*	$\mathbf{k}$	$i\mathbf{i}$	$i\mathbf{j}$	$i\mathbf{k}$*	i	1
$\mathbf{I}$	$\mathbf{J}$	$\mathbf{K}$	$i\mathbf{I}$	$i\mathbf{J}$	$i\mathbf{K}$		
$\mathbf{iI}$*	$\mathbf{iJ}$	$\mathbf{iK}$	$i\mathbf{iI}$	$i\mathbf{iJ}$	$i\mathbf{iK}$		
$\mathbf{jI}$*	$\mathbf{jJ}$	$\mathbf{jK}$	$i\mathbf{jI}$	$i\mathbf{jJ}$	$i\mathbf{jK}$		
$\mathbf{kI}$*	$\mathbf{kJ}$	$\mathbf{kK}$	$i\mathbf{kI}$	$i\mathbf{kJ}$	$i\mathbf{kK}$		

The introduction of symmetry-breaking

From our starting point with eight basic units, we find that the complete working out of all the possible combinations of vectors, scalars, pseudoscalars and quaternions, gives us 32 possible units or 64 if you have + and – signs. We have also seen that the group of order 64 requires only 5 generators. Though there are many ways of selecting these, all the pentad sets which incorporate all 8 starting units have the same overall structure (the only other set of generators which could produce all 64 elements being something like *i*, ***i***, ***j***, **i**, **j**). That is, the most efficient way of generating the 2 × 32 (and retain the 8 starting units) is to start with five *composites*, rather than eight primitives. That all such sets of 5 generators have the same pattern, we can see by splitting up the 64 units into 1, –1, *i* and – *i*, and 12 sets of 5 generators, each of which generates the entire group:

1	i				–1	$-i$			
$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{k}$	$\boldsymbol{j}$	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{k}$	$-\boldsymbol{j}$
$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{i}$	$\boldsymbol{k}$	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{j}$	$\boldsymbol{i}$	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$	$i\mathbf{k}$	$\mathbf{j}$	$-i\mathbf{i}\boldsymbol{i}$	$-i\mathbf{i}\boldsymbol{j}$	$-i\mathbf{i}\boldsymbol{k}$	$-i\mathbf{k}$	$-\mathbf{j}$
$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$	$i\mathbf{i}$	$\mathbf{k}$	$-i\mathbf{j}\boldsymbol{i}$	$-i\mathbf{j}\boldsymbol{j}$	$-i\mathbf{j}\boldsymbol{k}$	$-i\mathbf{i}$	$-\mathbf{k}$
$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{k}$	$i\mathbf{j}$	$\mathbf{i}$	$-i\mathbf{k}\boldsymbol{i}$	$-i\mathbf{k}\boldsymbol{j}$	$-i\mathbf{k}\boldsymbol{k}$	$-i\mathbf{j}$	$-\mathbf{i}$

The creation of any set of 5 generators of this form requires symmetry-breaking of one 3-D quantity. Starting with the perfect symmetry of

$$i \quad \mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \quad 1 \quad \boldsymbol{i} \quad \boldsymbol{j} \quad \boldsymbol{k}$$

we rearrange to produce:

i	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$	1
$\boldsymbol{i}$		$\boldsymbol{j}$		$\boldsymbol{k}$

Taking one of each of ***i j k*** onto each of the other three, we finally obtain:

$i\boldsymbol{k}$ $\quad \mathbf{i}\boldsymbol{i}$ $\quad \boldsymbol{i}\mathbf{j}$ $\quad \boldsymbol{i}\mathbf{k}$ $\quad 1\boldsymbol{j}$

Notably, we have to break the symmetry of one 'space' **i j k** or the other ***i j k***. Here, we choose ***i j k***. (We may note that set of generators of the form *i*, ***i***, ***j***, **i**, **j** would break the symmetry of both 'spaces' by privileging two dimensions of each. In the case of the space parameter, this would be in direct violation of its property of nonconservation, which includes rotation symmetry.)

The symmetry-breaking has physical consequences for the parameters involved. To create the generators we need to distribute the charge units onto the other parameters. So, from

Time	Space	Mass	Charge
i	**i j k**	1	***i j k***

we create the new 'compound' (and 'quantized') physical quantities, energy, momentum and rest mass:

i	$\mathbf{i}\boldsymbol{i}$ $\mathbf{j}\boldsymbol{i}$ $\mathbf{k}\boldsymbol{i}$	$1\boldsymbol{j}$
E	p_x p_y p_z	m
Energy	Momentum	Rest Mass

The combined object is identifiably nilpotent, squaring to zero, because

$$(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)\,(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m) = 0 \qquad (1)$$

which we can recognise as a version of Einstein's relativistic energy equation

$$E^2 - p^2 - m^2 = 0$$

or, in its more usual form,

$$E^2 - p^2c^2 - m^2c^4 = 0$$

Nilpotent quantum mechanics

To generate the Dirac equation we simply quantize the nilpotent equation, using differentials in time and space for E and p (Rowlands, 2005, 2007, 2010a, 2014, 2015). This is equivalent to simultaneously applying nonconservation and conservation. Einstein's relativistic energy equation (1) then becomes

$$\left(\mp \boldsymbol{k}\frac{\partial}{\partial t} \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m\right)\left(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m\right)e^{-i(Et-\mathbf{p.r})} = 0$$

Here, we have specified four sign variations in E and **p**. Nilpotency reduces this from eight, leading to another symmetry-breaking. The loss of a degree of freedom, involving $\pm m$, introduces chirality into the system. The four components, written out in full, are:

$(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin up	
$(i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin down	
$(-i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin down	
$(-i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin up	(2)

Since the signs of E and **p** intrinsically arbitrary, we identify the four possible states by adopting a convenient convention.

The spinor properties of the algebra are usually derived from a matrix representation, where the wavefunction is a 4-component spinor, incorporating fermion / antifermion and spin up / down states but still hold when expressed in the Clifford algebra form. These are easily identified with the arbitrary sign options for the iE and **p** (or **σ.p**) terms. In the nilpotent formalism we accommodate this by transforming $(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$ into a column vector with four sign combinations of iE and **p**, which may be written in abbreviated form as $(\pm\, i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$. Using the accepted convention, this can be either operator or amplitude. The symmetry between operator and amplitude is another leading to zero. The expression

$$(\pm\, i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m) \to 0$$

gives us both relativity and quantum mechanics, in a version which is much simpler and seemingly more powerful than conventional quantum mechanics. Here, we take the first bracket as an operator acting on a phase factor. The E and **p** terms can be covariant derivatives including any number of potentials or interactions with other particles. The Pauli exclusion principle follows immediately from nilpotent wavefunctions or amplitudes, because if any 2 particles are the same, their combination $\psi\psi$ is 0.

The nilpotent form, in fact, removes the need for a quantum mechanical equation of any kind. An operator of the form $(\pm\, i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$ will *uniquely* determine the phase factor needed to produce a nilpotent amplitude. So, rather than using a conventional form of the Dirac equation, we simply find the phase factor such that

$$(\text{operator acting on phase factor})^2 = \text{amplitude}^2 = 0.$$

If the defined operator has a more complicated form than that of the free particle, the phase factor will, of course, be no longer a simple exponential but the amplitude will still be a nilpotent.

The 5-fold generator structure ($i\boldsymbol{k}$, $\mathbf{i}\boldsymbol{i}$, $\mathbf{j}\boldsymbol{i}$, $\mathbf{k}\boldsymbol{i}$, $1\boldsymbol{j}$) also gives us the broken symmetry between the 3 charges:

$i\boldsymbol{k}$	$\mathbf{i}\boldsymbol{i}$ $\mathbf{j}\boldsymbol{i}$ $\mathbf{k}\boldsymbol{i}$	$1\boldsymbol{j}$
weak	strong	electric

which now adopt the characteristics of the mathematical objects they are connected to, and the corresponding group symmetries are a direct consequence, as can be demonstrated with full mathematical rigour:

pseudoscalar	vector	scalar
$SU(2)$	$SU(3)$	$U(1)$

There is even a version of the nilpotent operator and nilpotent Dirac equation that can be written for the *charges*, rather than for energy, momentum and rest mass or space and time derivatives, thus demonstrating that the charges form a basis for an equivalent ‘space’, to that of real space, which is ultimately that of vacuum.

The meaning of the dual group has also now become clear. By attaching quaternion operators to the time, space and mass terms, we have effectively exchanged real and imaginary terms, and the fourth term is provided by the spin angular momentum, which provides the same role in the quantized system as the overall charge structure. The first group effectively provides the entire ontology of physics, the second the means of observing it. The two groups together give us the quantized phase space of the fermion. In quantum mechanics, however, the two groups are not independent, since the second is derived from the first, and they do not commute. Ontology and epistemology are not independent. Also, the fourth term in the dual group (angular momentum) is not independent of the others. Ultimately the parameter group and its dual form ‘cancel’, not to zero, but to *h*.

Some other mappings and symmetries

There is also an octonion mapping of the 8 algebraic units of the 4 parameters, and to make it more exact we can take imaginary values of the spatial coordinates. Here, we see that the antiassociative parts of the multiplication table are those which have no physical meaning. It is as though antiassociativity were actually created to define the boundaries. In addition, group structure plays a key and defining role in both physics and the universal rewrite structure which we have described for all information systems, and antiassociativity prevents the octonions from being defined as a group.

*	1	***i***	***j***	***k***	***e***	***f***	***g***	***h***
1	*1*	***i***	***j***	***k***	***e***	***f***	***g***	***h***
I	***i***	*–1*	***k***	***–j***	***f***	***–e***	***–h***	***g***
J	***j***	***–k***	*–1*	***i***	***g***	***h***	***–e***	***–f***
K	***k***	***j***	***–i***	*–1*	***h***	***–g***	***f***	***–e***
E	***e***	***–f***	***–g***	***–h***	*–1*	***i***	***j***	***k***
F	***f***	***e***	***–h***	***g***	***–i***	*–1*	***–k***	***j***

G	***g***	***h***	***e***	***–f***	***–j***	***k***	*–1*	***–i***
H	***h***	***–g***	***f***	***e***	***–k***	***–j***	***i***	*–1*

*	*m*	*s*	*e*	*w*	*t*	*x*	*y*	*z*
M	*m*	*s*	*e*	*w*	*t*	*x*	*y*	*z*
S	*s*	*–m*	*w*	*–e*	*x*	*–t*	*–z*	*y*
E	*e*	*–w*	*–m*	*s*	*y*	*z*	*–t*	*–x*
W	*w*	*e*	*–s*	*–m*	*z*	*–y*	*x*	*–t*
T	*t*	*–x*	*–y*	*–z*	*–m*	*s*	*e*	*w*
X	*x*	*t*	*–z*	*y*	*–s*	*–m*	*–w*	*e*
Y	*y*	*z*	*t*	*–x*	*–e*	*w*	*–m*	*–s*
Z	*z*	*–y*	*x*	*t*	*–w*	*–e*	*s*	*–m*

The octonion structure is important in providing a basis for some higher groups such as E_8 which are considered to be significant in generating the spectrum of fundamental particles. While theories based on E_8 and other such groups normally have to introduce an arbitrary symmetry-breaking mechanism into their structures, here the brokenness is carried forward from the most basic level, in line with the principle that symmetry-breaking at this level has nothing to do with top-down 'mechanisms', but is really a combining into a higher symmetry of already disparate elements. The 8-fold basic unit combining those of space, time, mass and charge is already a 'broken' octonion.

In another significant algebraic representation, though the symmetry of the charge / vacuum space is broken in the generators of the 64-part algebra, a particular subalgebra of this larger algebra creates a symmetry between the two spaces which remains unbroken, and this has a particular physical significance. This is the H_4 algebra, derived by *coupling* the quaternions of the two spaces, with units 1, ***iI***, ***jJ***, ***kK***. The result is a cyclic but commutative algebra with multiplication rules

$$\boldsymbol{iI}\,\boldsymbol{iI} = \boldsymbol{jJ}\,\boldsymbol{jJ} = \boldsymbol{kK}\,\boldsymbol{kK} = 1$$
$$\boldsymbol{iI}\,\boldsymbol{jJ} = \boldsymbol{jJ}\,\boldsymbol{iI} = \boldsymbol{kK}$$
$$\boldsymbol{jJ}\,\boldsymbol{kK} = \boldsymbol{kK}\,\boldsymbol{jJ} = \boldsymbol{iI}$$
$$\boldsymbol{kK}\,\boldsymbol{iI} = \boldsymbol{iI}\,\boldsymbol{kK} = \boldsymbol{jJ}$$

The same result emerges if we couple the *negative* values of the paired vector units 1, –**iI**, –**jJ**, –**kK** (1 now being to –*ii*). This time we have:

$$(-\mathbf{iI})\,(-\mathbf{iI}) = (-\mathbf{jJ})\,(-\mathbf{jJ}) = (-\mathbf{kK})\,(-\mathbf{kK}) = 1$$
$$(-\mathbf{iI})\,(-\mathbf{jJ}) = (-\mathbf{jJ})\,(-\mathbf{iI}) = (-\mathbf{kK})$$
$$(-\mathbf{jJ})\,(-\mathbf{kK}) = (-\mathbf{kK})\,(-\mathbf{jJ}) = (-\mathbf{iI})$$
$$(-\mathbf{kK})\,(-\mathbf{iI}) = (-\mathbf{iI})\,(-\mathbf{kK}) = (-\mathbf{jJ})$$

If we now use the symbols $\mathsf{I} = \boldsymbol{iI} = -\,\mathbf{iI}$, $\mathsf{J} = \boldsymbol{jJ} = -\mathbf{jJ}$, $\mathsf{K} = \boldsymbol{kK} = -\,\mathbf{kK}$, 1, to represent this algebra, we can structure the relationships in a group table:

*	1	I	J	K
1	1	I	J	K
I	I	1	K	J
J	J	K	1	I
K	K	J	I	1

The group that emerges is a Klein-4 group, exactly isomorphous to the parameter group.

Now, all the standard aspects of spin and helicity can be easily recovered using nilpotent quantum mechanics. It is, therefore, possible to find a spinor structure which will generate the nilpotent quantum mechanical state vector, even though this process is not strictly necessary. A set of primitive idempotents constructing a spinor can be defined in terms of the H_4 algebra, incorporating the dual vector spaces:

$$\begin{gathered}(1 - \mathbf{iI} - \mathbf{jJ} - \mathbf{kK}) / 4 \\ (1 - \mathbf{iI} + \mathbf{jJ} + \mathbf{kK}) / 4 \\ (1 + \mathbf{iI} - \mathbf{jJ} + \mathbf{kK}) / 4 \\ (1 + \mathbf{iI} + \mathbf{jJ} - \mathbf{kK}) / 4 \end{gathered} \qquad (3)$$

The 4 terms add up, as required, to 1, and are orthogonal as well as idempotent, all products between them being 0. Essentially identical results can be generated using coupled quaternions rather than vectors:

$$\begin{gathered}(1 + \boldsymbol{iI} + \boldsymbol{jJ} + \boldsymbol{kK}) / 4 \\ (1 + \boldsymbol{iI} - \boldsymbol{jJ} - \boldsymbol{kK}) / 4 \\ (1 - \boldsymbol{iI} + \boldsymbol{jJ} - \boldsymbol{kK}) / 4 \\ (1 - \boldsymbol{iI} - \boldsymbol{jJ} + \boldsymbol{kK}) / 4 \end{gathered} \qquad (4)$$

The ‘spaces’ in the spinor structure are notably completely dual, neither being privileged with respect to the other. The orthogonality condition effectively creates a quartic space structure (a Finsler Berwald-Moor metric) with zero size, a point-particle (Rowlands, 2012). There is a significant chirality, however, in that, as in the nilpotent structure in (1), the signs cannot be completely reversed. Ultimately, when the spinors are applied to constructing the Dirac wavefunction in (1), this manifests itself in the positive sign of the m term.

If we reduce the 4-spinor expressions to a 2-spinor form, we remove the chirality. The chirality is a result of introducing the anticommuting symmetry of 3-dimensionality into a system based on complex numbers or equivalent, with the commutative symmetry of 2-dimensionality. So, using the quaternion version, the non-chiral 2-spinor form is becomes

$$(1 + \boldsymbol{iI}) / 2$$
$$(1 - \boldsymbol{iI}) / 2$$

which can be seen as equivalent to the more familiar projection operators

$$(1 - i\boldsymbol{i}) / 2$$
$$(1 + i\boldsymbol{i}) / 2$$

With $\boldsymbol{ij}$ for $i\boldsymbol{i}$, these are equivalent to $(1 - \gamma^5) / 2$ and $(1 + \gamma^5) / 2$. However, if we regard $\boldsymbol{i}$ and $\boldsymbol{i}$ as each part of a 3-D structure (as, ultimately, required by γ^5), then we can 'double' the complexity through dimensionalization and apply this to **p** (as in the universal rewrite system), making 2 × 2 into 3 + 1 and so introducing the chirality that we see in (3) and (4), as

$$(1 - \boldsymbol{jJ}) / 2 \qquad \text{and} \qquad (1 + \boldsymbol{jJ}) / 2$$

necessarily require the existence of

$$(1 - \boldsymbol{kK}) / 2 \qquad \text{and} \qquad (1 + \boldsymbol{kK}) / 2$$

Here, we see the fundamental difference between symmetries based on the number 2 and those based on the number 3. Uniqueness in Nature (which presupposes symmetry-breaking) frequently comes about through a 'competition' between these symmetries. We may note that the Weyl equation (the Dirac equation for massless particles, which applies to condensed matter pseudoparticles, but not to fundamental ones) effectively 'halves the wavefunction', eliminating the right-handed fermion and left-handed antifermion using these projection operators. Geometrically, the 2-component Pauli spinor is specified by a Möbius band, requiring a spatial twist; the 4-component Dirac spinor by a Klein bottle which is made of two Möbius bands.

By making the massless fermion one-handed, chirality is introduced even with the 2-spinor structure, but the chirality is yet another broken symmetry, because introducing a mass term also introduces a degree of the other handedness. The structure of the Dirac operator makes the chirality left-handed as opposed to the right-handed chirality of human beings (demonstrated by words such as *L sinister* for left-handed and *OE widdershins* for anticlockwise).

Vacuum

Another symmetry emerges from a looking at Pauli exclusion via a different perspective. If Nature corresponds to a totality of zero, and if we imagine creating a fermion, incorporating in the E and $\mathbf{p}$ terms all the potentials representing its interactions, in the form

$$(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

then the rest of the universe has to be structured so that it can be represented by

$$-\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

The nilpotent formalism requires a fermion to 'construct' its own vacuum, or the entire 'universe' in which it operates. The vacuum can be seen as 'delocalised' to the extent that the fermion is 'localised'. The nilpotency then defines the interaction between the localised fermionic state and the delocalised vacuum, with which it is uniquely self-dual, with the phase supplying he mechanism through which this is accomplished. In this case, we see that the often-made statement that quantum mechanics has not so far found a way of encompassing physics on a universal scale is very definitely untrue. Nilpotent quantum mechanics only works because we structure the entire universe at the same time as we structure any fermionic portion of it. This gives it the universality that has often been sought after using combinations with the supposedly more universal general relativity, in areas such as quantum gravity, loop quantum gravity and string theory. General relativity may provide a way of looking at physics on a universal scale and standard quantum mechanics of looking at physics on a local scale, but nilpotent quantum mechanics requires the simultaneous deployment of physics on both scales and in each case the results supersede those of the earlier, more partial theories.

Pauli exclusion also suggests that no two fermions can share the same vacuum. The 'hole' left by creating the particle from nothing in a particular state is the rest of the universe needed to maintain it in that state. This is what we mean by 'vacuum', and nilpotency ensures that the vacuum for one particle cannot be the vacuum for any other. The dual 'spaces' represented by **i j k** (real space) and **I J K** (vacuum space) effectively combine together to produce *zero totality* in a point particle with zero size. There is no other way of producing discrete points in space.

Fermion and vacuum also have a representation in terms of more abstract mathematics. Set boundaries themselves have vanishing boundaries, so the boundary of a boundary is zero:

$$\partial\partial = \partial^2 = 0$$

If A is a subspace of the entire space X, then the boundary ∂A is the intersection of the closures of A and of the complement of A or X – A, where the closure is defined as the union of the set and its boundary. If the universe is X, and the fermion A, the rest of the universe X – A. The point-fermion, of course, is itself a boundary, so the boundary of the fermion is 0. This is the meaning of nilpotency.

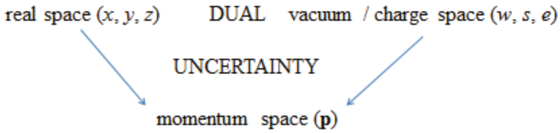
Nilpotency actually suggests multiple interpretations of vacuum and multiple specific results connected with it. The structures of the four components of the fermion in (2) show that two have $+E$ and two have $–E$. So the question arises: where are those with $–E$? The immediate answer is that they are in the vacuum space. There are just as many antifermions as fermions, but the chirality built into the structure, and that we can derive from the nilpotent or spinor structures, and even conventionally from the Dirac equation, means that only those in real space are observable.

Also deriving from our view of vacuum, the one-fermion theory of the universe (a modification of the one-electron theory of Stueckelberg-Wheeler-Feynman) becomes an increasingly attractive option. Here, the whole structure of the universe can be represented by a single fermion in an endless succession of backward and forward time states. The entire forward history of the universe will be contained in the fermion's vacuum or the rest of the universe associated with it, just as the localized state of the fermion is determined by its past. We can avoid determinism, however, because the fermion state can never be exactly defined. Also, the endless 'quantum' transitions that actually occur, either within a one-fermion or multiple-fermion state, and drive forward the arrow of time, may be seen as an expression of the endless bifurcations of the universal rewrite program based on totality zero that determines the information structure for the whole of Nature. The entropy then simply measures the number of bifurcations, say n, producing 2^n accessible states, and provides a measure of the perceived time that has passed, while the use of the maximum free energy in the minimum time is expressed in the principle of least action (Marcer and Rowlands, 2014a, b).

The one-fermion theory may be interestingly compared with a computer program. One fermion in many different states or manifestations is like the use of one symbol (1) in many different states in the computer program (these being determined by position with respect to the universal totality zero). The program can be seen as a single symbol in many manifestations, like the fermion describing the universe, where the backward and forward time states (or vacuum and localised fermion states) are also created by the fermion's relation to zero totality.

The idea that negative energy is essentially that of vacuum links it with gravity, which produces negative energy between identical masses compared with positive energy between identical charges. This means that gravitational energy can be considered as a kind of cancellation of the energies of the three gauge interactions, a gravity-gauge theory correspondence which was inherent in the present author's work long before it was adopted by string theorists. There is a further link, via the Dirac filled vacuum and the Higgs field, through the weak interaction. Because of the complex nature of the $i\mathbf{k}E$ term, the weak charge forms an effective dipole with the vacuum of opposite energy, with the spin of the fermion acting as a weak dipole moment. Dipoles, unlike monopoles, are attractive and so create negative energy.

Now, the symbol 1 for the fermion parallels the infinite string …11111111111111, representing –1 in binary, for vacuum, or vacuum space, alongside $-E$ and $-t$. This reflects the built-in bias for the fermion to be local and the antifermion nonlocal, though there are equal numbers of each. How, then can antifermions be local? This comes about because there are ways of *apparently* reversing time or making negative energy positive, which relate to the fact that, although real space and vacuum (charge space) are totally dual, neither of these space is totally dual with energy-momentum space, because the latter is partly constructed from each. Real or local antifermions and vacuum antifermions will then be a measure of how much this uncertainty affects the duality or complementarity of E, t and $\mathbf{p}$, $\mathbf{r}$.



To determine how many real, i.e. local, antifermions there are as a fraction compared to real fermions, we should look at processes such as *CP* violation, and the creation of neutrino masses, which would be unexpected in terms of pure charge considerations, but are certainly needed for neutrinos to be fermions. All the processes are related to the peculiarities of the weak interaction. The ratio of neutrino mass to the electroweak energy scale is about 10^{-12}. Something of a similar proportion occurs in $CP = T$ violation. In beta decay the mass factor disparity between nucleon and antineutrino is of order 7 billion, while baryon / antibaryon asymmetry is estimated from the photon / proton ratio of 10^9.

Vacuum reflections and partitions

The lead term in the fermionic column vector, representing the amplitude, defines the fermion type; the remaining terms are then equivalent to the lead term, subjected to the respective symmetry transformations, *P*, *T* and *C*, by pre- and post-multiplication by the quaternion units ***i***, ***j***, ***k*** which define the *vacuum space*:

Parity	P	$\boldsymbol{i}\,(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\,\boldsymbol{i} = (\pm\, \boldsymbol{i}kE \quad \mathbf{ip} + \boldsymbol{j}m)$
Time reversal	T	$\boldsymbol{k}\,(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\,\boldsymbol{k} = (\mp\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$
Charge conjugation	C	$-\boldsymbol{j}\,(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\,\boldsymbol{j} = (\mp\, \boldsymbol{i}kE \mp \mathbf{ip} + \boldsymbol{j}m)$

It follows immediately that $CP \equiv T$, $PT \equiv C$, and $CT \equiv P$ while $TCP \equiv\ CPT \equiv$ identity as

$$\boldsymbol{k}(-\boldsymbol{j}(\boldsymbol{i}(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\boldsymbol{k})\boldsymbol{j})\boldsymbol{j} = -\boldsymbol{kji}(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\boldsymbol{ijk} = (\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$$

The vacuum defined by the nilpotent formalism for each fermion state $(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$ is a continuous vacuum specified by $-(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$. This vacuum expresses the nonlocal aspect of the state. However, using the operators ***k***, ***i***, ***j*** we can partition the continuous state into discrete components with a dimensional structure. Idempotents become relevant in this context. If we postmultiply $(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$ by the idempotent $\boldsymbol{k}(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$ any number of times, the original state is reproduced except for a scalar multiple, which can be normalized away.

$$(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}(\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \to (\pm\, \boldsymbol{i}kE \pm \mathbf{ip} + \boldsymbol{j}m)$$

We can do the same for $\boldsymbol{i}(\boldsymbol{i}kE + \mathbf{ip} + \boldsymbol{j}m)$ and $\boldsymbol{j}(\boldsymbol{i}kE + \mathbf{ip} + \boldsymbol{j}m)$, except that the first of these produces an additional unit vector operator in alternate multiplications. This can also be normalized away, as it has no effect on the nilpotent structure of the wavefunction. We can, therefore, regard $\boldsymbol{k}(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$, $\boldsymbol{i}(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ and $\boldsymbol{j}(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ as vacuum operators and $(-i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$, $(i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$, and $(-i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$ as their respective vacuum 'reflections' at interfaces provided by *T*, *P* and *C* transformations, providing yet another insight into the meaning of the Dirac 4-spinor.

The three terms other than the lead term *in the spinor* can now be seen as the vacuum 'reflections' that are created along with the particle. The existence of the three vacuum operators as a result of partitioning the vacuum becomes a result of quantization and a consequence of the 3-part structure observed in the nilpotent fermionic state. The *zitterbewegung* then indicates that the vacuum is active in defining the fermionic state. Alongside their many other fundamental roles – as charges, *C*, *P*, *T* transformation operators, vacuum projections onto 3 axes, and indicators of fermion / antifermion / spin up / down in the Dirac spinor – the quaternion units $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ also constitute the dimensions of a vacuum space, dual to real space.

Zitterbewegung is an obvious manifestation of the duality between the two spaces, but the fermionic chirality means, in observational terms, it privileges the creation of positive rest mass. Thus the fermion has a half-integral spin because its creation requires a simultaneous division of the universe into two halves which are mirror images of each other at a fundamental level, but, at the observational level, they appear asymmetric because observation privileges the fermion singularity over vacuum, and real space over vacuum space.

The same *zitterbewegung* creates the invariant or rest mass of a fermion (or boson constructed from weak-charged fermions) by switching between real and vacuum space at the Compton frequency. The combination of the dual spaces (or space and antispace) creates the only thing in nature with zero size, the point-charge or point-like fermion. In general, energy cannot be observed at a point except in the form of rest mass, and so only particles with point-charges (of some kind) can be manifested as points with a finite mass. That is, invariant mass is a property only of point-charges or point-like particles. It is notable, for example, that the flow of energy in the electromagnetic field given by the Poynting vector cannot be realized at a point. The types of energy which are not point-like can be seen as the manifestation of the interactions of point-particles with the vacuum, that is, other point-particles or 'the rest of the universe'.

The Higgs field is, remarkably, a field of constant energy manifested at every conceivable point in space in an absolute continuum. It is only by coupling with this field that the charged fermions, or bosons constructed from them, gain mass. We can imagine it as a potential source of point-particle creation at every conceivable spatial point. It may be compared with the Newtonian idea of absolute space as an unbroken continuum in which observable points exist as manifestations of observable Euclidean space produced by charges.

One special case of vacuum reflection is of particular interest, as it is concerned with a very familiar observation. Reflection in a real mirror is due to an aspect of the electric force. The laterally-inverted virtual image produced by a mirror is actually due to the rest of the universe (vacuum, in our terminology) of which the mirror is a component. The virtual image is the reflection due to one component force: the electric component created by $\boldsymbol{j}$. The mirror is constructed physically to concentrate the resources of vacuum almost entirely on this single force and to cancel all other contributions.

Symmetries applied to fundamental particles

The pattern of double 3-dimensionality emerging from duality and anticommutativity and leading to broken symmetry at order 5 is very apparent in biology as work done with Vanessa Hill testifies (Hill and Rowlands, 2010a,b), and work done with Peter Marcer suggests that it underlies self-governing systems in general (Marcer and Rowlands, 2014a,b). We have traced the pattern and its mathematical origin in zero totality and find the same numbers and characteristic consequences repeat for Platonic and Archimidean solids (in any number of dimensions), kissing numbers, algebraic equations, quantum mechanics, fundamental particles, the periodic table, DNA / RNA, higher biological structures, etc. The 5-fold pattern is the one that links lower order systems with higher order ones. This is especially apparent in biology, where the 5-fold structure can be seen emerging in the structures in both downward and upward directions.

Multiples of 2 and 3 occur over and over again in these structures, and, where they do, the dualistic and anticommutative origins can be established. Where 5 occurs it is always due to a broken symmetry, and the emergence of 5 can be seen as the key factor in the emergence of something new. Groups like E_8 are entirely constructed from such units. As illustration we could take some structures related to fundamental particles.

If we look at the fundamental particles, all the symmetries which apply to them seem to be constructed from smaller symmetries based on these units. The same also applies to many of the groups thought to be of significance in this area, particularly those based on the octonion symmetries, such as the exceptional groups E_6, E_7 and E_8. Because the symmetry-breaking is ultimately 3-dimensional in origin (and manifested, for example, in quarks and 3 particle generations), the symmetries involved in particle groupings tend to map naturally onto geometries in 3-dimensional space. Among various possible representations, the quark / lepton structures can be associated naturally with the 12 pentads of the Dirac algebra:

generation		*isospin*					
1	up quark	up	$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{k}$	$\boldsymbol{j}$
	down quark	down	$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$	$i\mathbf{k}$	$\mathbf{j}$
2	charm quark	up	$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{i}$	$\boldsymbol{k}$
	strange quark	down	$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$	$i\mathbf{i}$	$\mathbf{k}$
3	top quark	up	$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{j}$	$\boldsymbol{i}$
	bottom quark	down	$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{k}$	$i\mathbf{j}$	$\mathbf{i}$
1	antiup-quark	up	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{k}$	$-\boldsymbol{j}$
	antidown-quark	down	$-i\mathbf{i}\boldsymbol{i}$	$-i\mathbf{i}\boldsymbol{j}$	$-i\mathbf{i}\boldsymbol{k}$	$-i\mathbf{k}$	$-\mathbf{j}$
2	anticharm-quark	up	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{i}$	$-\boldsymbol{k}$
	antistrange-quark	down	$-i\mathbf{j}\boldsymbol{i}$	$-i\mathbf{j}\boldsymbol{j}$	$-i\mathbf{j}\boldsymbol{k}$	$-i\mathbf{i}$	$-\mathbf{k}$
3	antitop-quark	up	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
	antibottom-quark	down	$-i\mathbf{k}\boldsymbol{i}$	$-i\mathbf{k}\boldsymbol{j}$	$-i\mathbf{k}\boldsymbol{k}$	$-i\mathbf{j}$	$-\mathbf{i}$

Another structure is especially interesting as it was created to explain fundamental particles using the E_8 symmetry, and algebra (Rowlands, 2008), but, as Vanessa and I showed, clearly applies on a massive scale to geometrical, chemical and biological structures (Hill and Rowlands, 2010a,b). The symmetries connected with fundamental particles seem invariably to be constructed from smaller symmetries based on the units 2, 3 and 5. The same principle applies to the group structures which are of significance in this field, and especially those which are generated by the octonion symmetries, such as the exceptional groups E_6, E_7 and E_8. The symmetry-breaking in these cases originates in 3-dimensionality (for example, in quarks and 3 particle generations), and the symmetries which group the various particle structures have a tendency to connect with geometries in described 3-dimensional space.

Various authors have suspected that the largest exceptional group E_8 might be the group which unifies the fundamental particles, and the possibility was discussed in *Zero to Infinity* and other places. In a paper which received a lot of publicity at the time, A. Garrett Lisi (2007) proposed that all the known fermions *and* the gauge bosons could be described by the 240 root vectors of the E_8 group. There are many problems with this model. The number of particles listed is not 240, leading to a completely *ad hoc* speculation by Lisi about 'missing' ones. There is no natural explanation of the generations, and the gravity theory is arbitrary and speculative. The assignments are often unnatural and hard to understand. Yet the idea of using E_8, and especially the 240 root vectors, is not unreasonable and much more convincing assignments could be made. Consider, for instance, the following table of quarks, leptons and gauge bosons:

	quarks	*leptons*	*bosons*		*fermions*	*bosons*							
1	3	1	1	=	4	1	=	5					
2	6	2	2	=	8	2	=	10	S				
3	9	3	3	=	12	3	=	15					G
4	12	4	4	=	16	4	=	20	S	I			
5	18	6	6	=	24	6	=	30	S				G
6	24	8	8	=	32	8	=	40	S	I	A		
7	36	12	12	=	48	12	=	60	S	I			G
8	48	16	16	=	64	16	=	80	S	I	A	V	
9	72	24	24	=	96	24	=	120	S	I	A		G
10	144	48	48	=	192	48	=	240	S	I	A	V	G

The table is based only on duality (2), anticommutativity (3) and symmetry-breaking (5), derived ultimately from the 2 / 3 combination. There are four factor 2 dualities, based on the four fundamental parameters space, charge, time and mass: spin up / down (S), isospin up / down (I), fermion / antifermion (A), particle / vacuum (V); and there is a single factor 3 triplet, representing the generations (G), which comes from *CPT* symmetry. The basic structure of each row is a 5, consisting of 3 quarks + 1 lepton + 1 boson (row 1), to which the four factor 2 dualities and single factor 3 triplet are then applied in every conceivable combination to rows 2-10. The 5 of each row is notably an artificial construct, which links fermions with bosons in exactly the same way as a fermionic nilpotent is constructed. According to Lisi, the fermion-boson link is possible in the exceptional groups E_6, E_7 and E_8, though not in lower order ones. In our understanding, it is possible because the last term in a nilpotent structure can be a scalar. Excluded from the structure are the spin 0 Higgs boson, which is not a gauge boson and the spin 2 graviton, which has never been shown to exist. The spin 1 inertial pseudoboson, which we have postulated elsewhere as the 25th term needed for Grand Unification, is not really a separate particle from the photon, but rather a special realisation of it at the Planck energy (Rowlands, 1992, Rowlands and Cullerne, 2002).

In deriving this structure, we have invoked the universal rewrite principle in which pentad units can be created either by an upward or downward movement from the previous level, so allowing the same structures to emerge fractally at every level in the process. We have inverted the derivation of 12 structures from a 5-unit pentad in our table of quarks / leptons, and mapped the fermions and bosons onto a new pentad structure. The pseudoscalar component of this (the *iE* term in the pentad) is 24 leptons / antileptons, and the vector component (the **p** term) 72 quarks / antiquarks. Bosons, of course, are scalar particles, and the squared products of fermions / antifermions, just as scalars are the squared products of pseudoscalars and vectors. So by making the 24 bosons occupy the *scalar* part of the pentad (the *m* term), we can use nilpotency to group the 96 fermions (24 leptons and 72 quarks) with the 24 bosons into a single structure with 120 fermions plus bosons. The stages in this process would seem to be represented by the combinations 48 + 12 = 60, 96 + 24 = 120, 192 + 48 = 240.

Conclusion

A hierarchy of symmetries can be seen to arise from the Klein-4 symmetry between mass, time, charge and space. This is the most fundamental symmetry in physics. It can be used directly to generate many physical laws and principles, and its representation in algebraic units leads to a version of relativistic quantum mechanics which is applicable to the fundamental particle or fermionic state. The group structure of the algebra also generates the symmetry-breaking between the interactions which occurs at the most fundamental level in physics. Many symmetries which occur at the deepest levels in physics can be seen to be consequences of this one.

Appendix: The Coupling Constants

The coupling constants for the 3 gauge interactions ‘run’ with different energies of interaction (*m*). These are given by standard formulae, but I have previously modified the first for quarks with integral charges (Rowlands, 2007, 2014).

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha_G} + \frac{3}{\pi} ln \frac{M_X^2}{\mu^2} .$$

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G} - \frac{5}{6\pi} ln \frac{M_X^2}{\mu^2}$$

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_G} - \frac{7}{4\pi} ln \frac{M_X^2}{\mu^2}$$

$$\sin^2 \theta_W = \frac{\alpha(\mu)}{\alpha_2(\mu)}$$

As before, we have 4 equations and 6 unknowns. For each value of *m*, we need α, α_3, α_2, the grand unified coupling constant α_G, the grand unified mass M_X, and $\sin^2\theta_W$. Working out the equations, with $\sin^2\theta_W = 0.25$ unifies all three coupling constants exactly at the Planck mass with $\alpha_G = 1 / 52.4$. However, it is always worth trying out variations on a theme, even if the alternatives seem less likely. We have observed how α_3 becomes something like 1 /8 at $\mu = 60$ GeV, and stays reasonably close to this value for energies within the electroweak scale (80.2 to 246 GeV). This would then be comparable with $\alpha_2 = 1 / 32$ and $\alpha = 1 /1 28$ at these energies.

These coupling coefficients, when multiplied by the square of the 'Planck charge' $(\hbar c)$ give us the charge squared values for the strong, electric and weak interactions at these energies, which we could write as: $(1/2^2)$ $(\hbar c)$ / 2, $(1/2^4)$ $(\hbar c)$ / 2, $(1/2^6)$ $(\hbar c)$ / 2. However, there is no definite reason to choose $(\hbar c)$ as the fundamental unit of charge squared, and equivalently, $G \times$ fundamental unit of mass squared in 'quantum gravity', rather than, say, $(\hbar c)$ / 2. If we choose $(\hbar c)$ / 2, then we can form a set of ratios for the sources of gravitational, strong, weak and electric interactions approximating to $1/2^0$, $1/2^1$, $1/2^2$ and $1/2^3$.

Maybe this is no more than numerology, but the reductions seem to follow the degrees of specification which the charges introduce and the progression from vector to pseudoscalar to scalar. They also reflect the same, as applied to angular momentum conservation. The strong charge incorporates information about magnitude, handedness and direction, the weak charge incorporates magnitude and handedness, and the electric charge magnitude only, with corresponding reductions in strength. The source of gravity has no reductions because it has no quaternionic charge structure, with + and – values, and so does not even have the 1/2 reduction of the strong charge.

If we follow the logic of using $(\hbar c)$ / 2, and remember that these calculations are only good to first order, then we would need to replace $M_X = M_P$ with $M_X = M_P / \sqrt{2}$. Essentially, then, α_G becomes 1 / 52, $\alpha \approx 1 / 127$ at the electroweak scale, with $\alpha_2 \approx 1 / 31$, while $\alpha_3 = 0.5$ (the new ideal value), occurs at 72 MeV, which is close to the assumed 'fundamental mass' $m_f = m_e / \alpha = 70$ MeV; α_3 becomes 0.33 at 1 GeV, which is close to the observed value.

References

Hestenes, D. 1966, *Space-Time Algebras, Gordon and Breach*, New York Rowlands, P., 1979, *The Laws of Physics and Fundamental Particles*, NVA Ltd.

Hill, V. J. and Rowlands, P., 2010a, Nature's Fundamental Symmetry Breaking, *International Journal of Computing Anticipatory Systems*, **25**, 144-159, 2010

Hill, V. J. and Rowlands, P., 2010b, The Numbers of Nature's Code, *International Journal of Computing Anticipatory Systems*, **25**, 160-175, 2010

Lisi, A. Garrett, 2007, An Exceptionally Simple Theory of Everything, arXiv:0711.0770

Marcer, P. J. and Rowlands, P., 2014a, Information, Bifurcation and Entropy in the Universal Rewrite System, *International Journal of Computing Anticipatory Systems*, 27, 203-215

Marcer, P. J. and Rowlands, P., 2014b, Universal Rewrite and Self-Organization, with P. Marcer, *International Journal of Computing Anticipatory Systems*, 27, 227-246

Rowlands, P., 1982, The fundamental parameters of physics, ASIS/NAPS microfiche 04068.

Rowlands, P., 1983, The fundamental parameters of physics. *Speculat. Sci. Tech.*, **6**, 69-80

Rowlands, P., 1991, *The Fundamental Parameters of Physics: An Approach towards a Unified Theory*, PD Publications, Liverpool

Rowlands, P., 1992, Is the gravitational field really nonlinear? *Proceedings of Conference on Physical Interpretations of Relativity Theory III*, British Society for Philosophy of Science, London, 311-318.

Rowlands, P., 2001, A foundational approach to physics, arXiv:physics/0106054

Rowlands, P., 2002, Representations of a Fundamental Theory, in K. Bowden (ed.), *Boundaries*, May 2003 (*Proceedings of XXIV ANPA Conference*, Cambridge, August 2002), 18-47

Rowlands, P., 2005, Removing redundancy in relativistic quantum mechanics, arXiv.org:physics/0507188

Rowlands, P. 2007, *Zero to Infinity The Foundations of Physics*, Singapore, London and Hackensack, NJ: World Scientific

Rowlands, P., 2008, A minimalist approach to quantum mechanics and particle physics, K. Bowden and A. D. Ford (eds.), *Exactness*, 252-266

Rowlands, P. 2009, Are there alternatives to our present theories of physical reality?, arXiv:0912.3433

Rowlands, P., 2010a, Physical Interpretations of Nilpotent Quantum Mechanics, arXiv: 1004.1523

Rowlands, P., 2010b, Mathematics and Physics as Emergent Aspects of a Universal Rewrite System, *International Journal of Computing Anticipatory Systems*, **25**, 115-131, 2010

Rowlands, P., 2012, A null Berwald-Moor metric in nilpotent spinor space, *Symmetry*, 23, no. 2, 179-188, 2012

Rowlands, P., 2013, Space and Antispace, in R. L. Amoroso, L. H. Kauffman and P. Rowlands (eds.), *The Physics of Reality Space, Time, Matter, Cosmos*, World Scientific, 29-37

Rowlands, P. 2014, *The Foundations of Physical Law*, Singapore, London and Hackensack, NJ: World Scientific

Rowlands, P. 2015, *How Schrödinger's Cat Escaped the Box*, Singapore, London and Hackensack, NJ: World Scientific

Rowlands, P. and Cullerne, J. P., 2002, Can gravity be included in grand unification?, in R. L. Amoroso, G. Hunter, M. Kafatos and J-P. Vigier (eds.), *Gravitation and Cosmology: From the Hubble Radius to the Planck Scale*, Kluwer, 279-86

Rowlands, P. and Diaz, B., A universal alphabet and rewrite system, arXiv:cs.OH/0209026, with B. Diaz