3-dimensionality

Peter Rowlands

Department of Physics, University of Liverpool, Oliver Lodge Laboratory, Oxford St, Liverpool, L69 7ZE, UK. email p.rowlands@liverpool.ac.uk

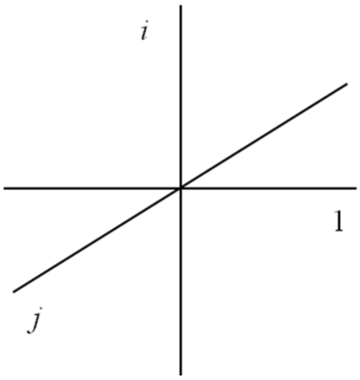
Abstract. 3-dimensionality is one of the most important and profound ideas in the whole of physics. The fact that space is 3-dimensional gave us a special insight into Nature from an early period because it set us the problem of explaining why this counterintuitive structure was so prevalent in natural systems. Part of the answer was revealed by the discovery of quaternions by Hamilton in 1843, although this spectacular insight was not followed up with the thoroughness it deserved. Since then we have found that Nature requires two 3-dimensionalities to define all the important structures in physics, chemistry and biology, as well as those of algebra and geometry. Here, we will largely discuss the importance of 3-dimensionality in physics.

Introduction

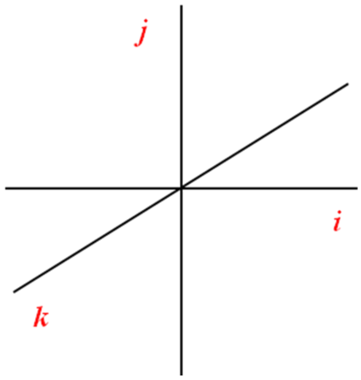
3-dimensionality is one of the most important concepts in physics. It is also one of the most important breakthroughs in the whole of human knowledge. It presents us with an extraordinary and non-obvious fact which challenges interpretation. That space is 3-dimensional has probably been known in some form for millennia. But, as we shall see, the concept extends beyond space, and the spatial example leads us to other ones which are equally important in physics. In addition to the fact that 3-dimensionality has such a special and peculiar signature that it provides one of the main access routes to unravelling Nature's most fundamental secrets, it is also important for many other reasons. As I hope to show, it is the only source for discreteness and localisation in Nature. Without it, measurement and observation would be impossible. And it appears in many guises. It is a ubiquitous condition that extends far beyond its appearance in the structure of space.

Though it is fashionable to be cavalier about dimensions and to act as though they can be invented and removed at will – as well as distorted and modified – to explain aspects of physics, it is an inescapable fact that space, the only physical parameter that gives us a direct way of appreciating the physical world through measurement, is most definitely 3-dimensional. And this 3-dimensionality cannot be extended to, say, 9-D, though it could be embedded in a higher dimensionality including other things.

Much has been said about the difficulty of explaining 3-dimensionality, but, in fact, it isn't difficult at all. We have known the reason why 3-dimensionality is special for more than 150 years. This happened when Sir William Rowan Hamilton was trying to extend the '2-D' complex number system to explain 3-D space.



‘triads’



quaternions

Hamilton’s attempted extension of the Argand diagram to include a second imaginary axis (j) in a system of ‘triads’ failed because there was no meaning for the product $\boldsymbol{ij}$ within the system. There was no closure in the algebra, but eventually by positing three cyclic imaginary dimensions ($\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$) alongside the real one (which could not be shown on the diagram), he was able to generate a division algebra that was closed and consistent.

The need for a real product means that the 3-D quaternions are in some way part of a larger 4-dimensionality as their name implies. You can’t even define the 3 imaginary units without invoking the real one (1), and each of the 3 imaginary axes is in some sense orthogonal to the real one that cannot be drawn on the same diagram. But this one is not like the other 3. It cannot be totally absorbed into 3 axes, which are effectively, in the pure mathematics, interchangeable with each other. The relationship is a broken one like the real and imaginary axes of complex numbers.

Quaternions, significantly, are anticommutative, with the multiplication rules:

$$\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}; \qquad \boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}; \qquad \boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}; \qquad \boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = -\boldsymbol{ijk} = 1.$$

This becomes apparent when we multiply $\boldsymbol{ij}$ by $\boldsymbol{ji}$, for

$$\boldsymbol{ij}\,\boldsymbol{ji} = -\,\boldsymbol{ii} = 1$$

not –1. The significance of this is that it explains 3-dimensionality, for $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{ij}$ form a closed set, which cannot be extended (unless antiassociativity is additionally introduced – and then only in the singular case of octonions). It also leads to a cross-product, which is characteristic of 3-, but not higher, dimensionality. In simple terms, you can’t make anticommutativity work if you have more than 3 terms. In the absence of antiassociativity,

$$\text{anticommutativity} \equiv \text{3-dimensionality}$$

The ‘3-ness’ is thus not the primary cause of the 3-dimensionality of space. It is simply a result of anticommutativity. If we have two axes, $\boldsymbol{i}$ and $\boldsymbol{j}$, that are anticommutative with each other, then we cannot draw any other axis that is anticommutative with them, unless it is $\boldsymbol{ij}$, which we also call $\boldsymbol{k}$. Anticommutativity forces 3-dimensionality. The strange arbitrariness of the number 3 is explained. Commutative things, of course, can be defined to infinity. If $\boldsymbol{i}$ and $\boldsymbol{j}$ were commutative, we could have $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, $\boldsymbol{l}$, $\boldsymbol{m}$, etc. without limit. In physical terms, anticommutative things ‘know’ about each other’s presence and have to act accordingly; commutative things do not.

Quaternions are known as one of the four division algebras in mathematics. The only higher dimensional one is octonions with 7 imaginary components and one real, compared to the 3 imaginary and one real of quaternions, and octonions are antiassociative as well as anticommutative. So, for example, $a(bc) = -(ab)c$.

real numbers	1	commutative	associative
complex numbers	2	commutative	associative
quaternions	4	anticommutative	associative
octonions	8	anticommutative	antiassociative

Significantly, the kind of dimensionality found in space, which is characteristic of that of a *nonconserved* quantity, with an affine structure and rotation symmetry between the dimensions, can only exist for 3 dimensions unless the system is also antiassociative. Even then it can only exist for 7 dimensions. *There is no extension of space as we know it to 9 dimensions.* The 3 dimensions of space could be *embedded* in a higher dimensionality but they cannot be extended to become one. It is mathematically impossible. And the application of division algebras to space is crucial for physical purposes, where measurement and counting are fundamental processes. (Rowlands, 2015c)

The algebra of space

Quaternions are the only *pure* 3-D system. The 3 cyclic products produce each other and nothing else. But space is *not* pure 3-D. We can go back to Hamilton to find out why. Hamilton early on decided to extend quaternions by complexifying them. So he took the product

$$(1, i) \times (1, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$$

where i is the ordinary (complex) square root of -1. Ordinary complex numbers are, of course, distinct from quaternions in being commutative. Our base set is now 1, i, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, and, multiplying everything out, we will also generate terms like $i\boldsymbol{i}$, $i\boldsymbol{j}$, $i\boldsymbol{k}$. For reasons that will soon become clear, we will write $i\boldsymbol{i} = \mathbf{i}$, $i\boldsymbol{j} = \mathbf{j}$, $i\boldsymbol{k} = \mathbf{k}$.

If we take the products of these terms, we can write:

$$(i\boldsymbol{i})^2 = (i\boldsymbol{j})^2 = (i\boldsymbol{k})^2 = -i(i\boldsymbol{i})\,(i\boldsymbol{j})\,(i\boldsymbol{k}) = 1$$
$$(i\boldsymbol{i})\,(i\boldsymbol{j}) = i(i\boldsymbol{k})$$
$$(i\boldsymbol{k})\,(i\boldsymbol{i}) = i(i\boldsymbol{j})$$
$$(i\boldsymbol{j})\,(i\boldsymbol{k}) = i(i\boldsymbol{i})$$

We can also write these relations in the form:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -i\mathbf{ijk} = 1$$
$$\mathbf{ij} = i\mathbf{k}$$
$$\mathbf{ki} = i\mathbf{j}$$
$$\mathbf{jk} = i\mathbf{i}$$

The complexified quaternion units $i\boldsymbol{i} = \mathbf{i}$, $i\boldsymbol{j} = \mathbf{j}$, $i\boldsymbol{k} = \mathbf{k}$ are, of course, anticommutative in exactly the same way as ordinary quaternion units, but we now notice an extra feature, the i term outside the bracket that has appeared on the right-hand side of the equations. The terms we have been using here, $i\boldsymbol{i} = \mathbf{i}$, $i\boldsymbol{j} = \mathbf{j}$, $i\boldsymbol{k} = \mathbf{k}$, are easily identifiable as the units of the Clifford algebra of 3-D space. They are also called multivariate vector units (Hestenes, 1966) and are isomorphic to the Pauli spin matrices. I will refer to them simply as vector units.

They are the true algebra of 3-D space, but they are not a pure 3-D algebra, because the products of $\mathbf{ij}$, etc., are not themselves vector units but *pseudovectors*, such as $i\mathbf{k}$. These are mathematically the same as quaternions and are seen in physical concepts such as area and angular momentum. In addition the triple product $\mathbf{ijk}$ is a pseudoscalar i, as with volume, if we use pure space units.

In principle, the full algebra of vector space requires us to use a set of 8 base units, which includes

vectors	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$
pseudovectors = quaternions	$i\mathbf{i}$, $i\mathbf{j}$, $i\mathbf{k}$
pseudoscalar = complex term	i
scalar	1

Including + and – values, this requires 16 base units, as opposed to 8 for quaternions and 4 for complex numbers. We also note that quaternions, complex numbers and real (scalar) numbers are subalgebras of this one.

The fact that space isn't a 'pure' 3-D means that complex numbers have to be taken into account when we are discussing its properties, perhaps also explaining how a complex variable like time can be absorbed relatively naturally into the '4-vector' combination of space and time. If we take the complexified version of the quaternion base units 1, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, we naturally produce the 4-vector units i, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Even though the i term here refers in the first instance to volume, rather than time, the commutativity of the relation between i and the quaternion units means that the physical nature of the unit doesn't need to be specified.

The existence of a complexified quaternion system for describing one of the fundamental physical parameters suggests the question, never fully answered: Is there any role for pure quaternions at the fundamental level in physics? Well, we have had 150 years of extraordinarily vituperative anti-quaternion propaganda obscuring the great simple fact that quaternions have solved the problem of 3-dimensionality. Even if the absurdity of this is conceded, there is always the fallback position that quaternions have never found any physical role of their own.

There is no doubt that the quaternion system underpins the 3-D properties of space, and this has been emphasized by the subsequent discoveries of quantum mechanical spin and the special relativistic connection of space and time, both of which it can be said to have predicted. However, it would seem strange if such a perfect system should only have such an indirect manifestation. Do quaternions have any more direct manifestation in physics?

This is where we need to introduce the Klein-4 parameter group, which I have discussed many times previously. (Rowlands, 1983, 1991, 2001, 2007, 2009, 2014, 2015a) There is no doubt that the quaternion system underpins the 3-D properties of space, and this has been emphasized by the subsequent discoveries of quantum mechanical spin and the special relativistic connection of space and time, both of which it effectively predicted.

A group of order 4

The basis of this is that Nature exhibits a zero totality, which, at the fundamental level, is manifested through a symmetry between the parameters space, time, mass and charge. The structure is that of a Klein-4 or D_2 group which can be represented in many different ways. Strikingly, it is full of manifestations of 3-dimensionality or anticommutativity, in addition to numerous dualities or summations to zero.

mass	conserved	real	continuous (1-D) commutative
time	nonconserved	imaginary	continuous (1-D) commutative
charge	conserved	imaginary	discrete (3-D) anticommutative
space	nonconserved	real	discrete (3-D) anticommutative

In a more extensive specification, the properties / antiproperties would become:

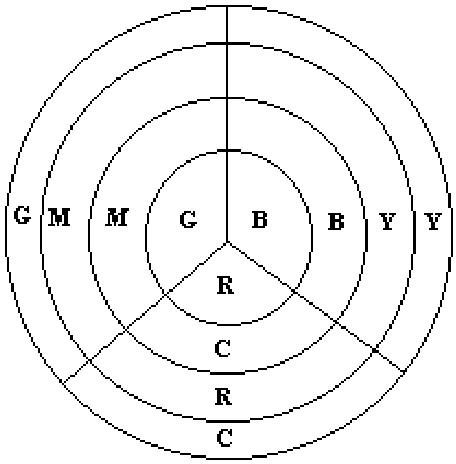
mass	conserved identity translation asymmetric	real norm 1	continuous nondimensional global commutative
time	nonconserved no identity translation symmetric	imaginary norm –1	continuous nondimensional global commutative
charge	conserved identity translation asymmetric rotation asymmetric	imaginary norm –1	discrete dimensional local anticommutative
space	nonconserved no identity translation symmetric rotation symmetric	real norm 1	discrete dimensional local anticommutative

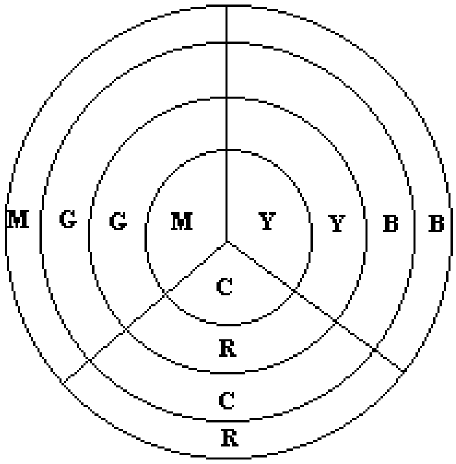
Using the symbols, *x*, *y* and *z*, to represent the properties, with –*x*, –*y* and –*z* standing for the exactly opposite ‘antiproperties’, indicates that this symmetry incorporates a conceptual zero:

mass	x	y	z
time	$-x$	$-y$	z
charge	x	$-y$	$-z$
space	$-x$	y	$-z$

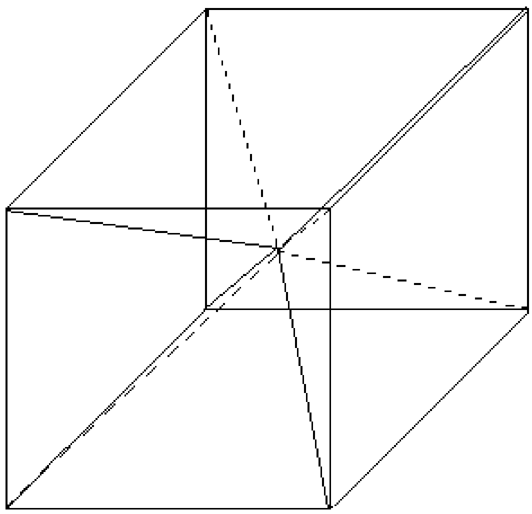
The 3-dimensionality inherent in this structure is inherent in the use of 3 properties and 3 symbols, *x*, *y* and *z*, and can be seen in the use of 3-dimensional representations, and an analogous use of 3 colours. (Rowlands, 2003, 2007, 2014, 2015a)

The colour representation uses the 3 primary colours (R, G, B) as properties and the 3 secondary colours (C, M, Y) as antiproperties or vice versa, with colourless (white) summation of each sector. Concentric circles represent the parameters in any order.

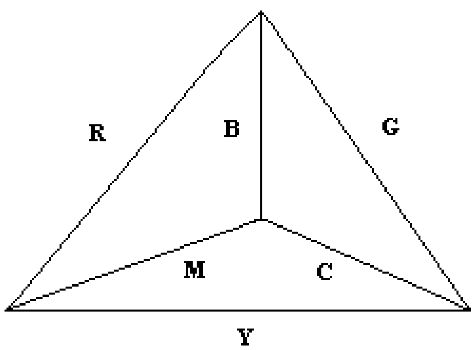




The 3-D representation plots the properties as x, y, z and the antiproperties as $-x$, $-y$, $-z$ to four corners of a cube from its centre. The dual group formed by reversing one property / antiproperty combination is represented by four dotted lines.



The tetrahedral representation uses the primary and secondary colours as in the colour representation. The four parameters appear as either the four faces or four vertices, with the edges representing the six properties / antiproperties.

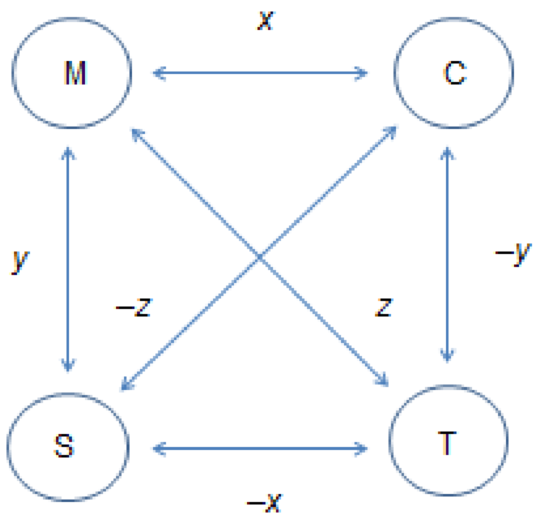


Now, both the set of parameters, and consequently all of physics, can be reduced to algebra, because the characteristics of the parameters can be defined entirely through the algebras that represent them.

mass	scalar	1
time	pseudoscalar (complex)	*i*
charge	quaternion (pseudovector)	***i***, ***j***, ***k***
space	vector	**i**, **j**, **k**

This is a kind of ontological ordering because the complexity increases at each level. Mass(-energy), time and space have the algebras that we familiarly associate with them, or that, in the case of time, are derived from developments in physics.

Yet another representation is a 'graph' which also suggests an interpretation in category theory, which seems to apply to both the parameter group of mass, charge, space and time, and the underlying universal rewrite system on which it is based. This representation again makes use of the algebraic representation of properties and antiproperties:



A related diagram was drawn by Vanessa Hill for my video-recorded lectures on *The Foundations of Physical Law*, based on a rough sketch provided by myself.

The algebra of charge

The algebra that is least familiar in this set is that of charge, which fills what would otherwise be a gap in the ontological sequence. The reason why this is less familiar is because the concept of 'charge' has only been slowly extended to incorporate the sources of strong and weak, as well as electrical, interactions. I can certainly remember this usage, now so relatively familiar, being met with incomprehension thirty and even twenty years ago. Even so, the use of the word is to mean a simple fundamental thing is still somewhat equivocal.

Wikipedia, for example, says that: ‘Various charge quantum numbers have been introduced by theories of particle physics. These include the charges of the Standard Model: The color charge of quarks. The color charge generates the $SU(3)$ color symmetry of quantum chromodynamics. The weak isospin quantum numbers of the electroweak interaction. It generates the $SU(2)$ part of the electroweak $SU(2) \times U(1)$ symmetry. Weak isospin is a local symmetry, whose gauge bosons are the W and Z bosons. The electric charge for electromagnetic interactions. In mathematics texts, this is sometimes referred to as the u_1-charge of a Lie algebra module.’ The article also says that, in gravitation, charges are ‘Eigenvalues of the energy-momentum tensor correspond to physical mass.’

The basic idea is that ‘charges’ are the sources of the four known fundamental interactions. However, the idea that there is some common structure is never stated explicitly and the charge concept is only introduced in roundabout ways, and seems a little confused, especially in its lack of reference to the coupling constants for the 3 interactions.

Why is this? The simple answer is that the four known interactions are based on different symmetry groups and have different physical manifestations, and that, in addition, there are fundamental differences between gravity and the other 3 (gauge) interactions.

The problem might be considered insuperable but for two things:

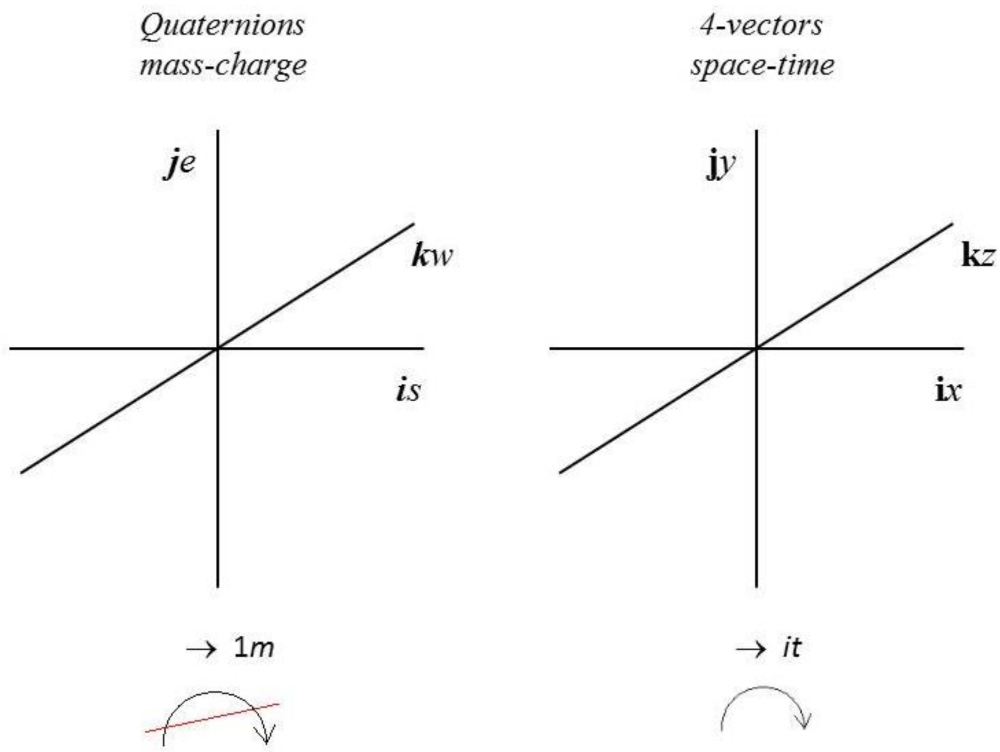
(1) Whatever extra complications are manifested, all four forces have a ‘Coulomb’ or inverse square component, and that, in the case of gravity, it is attractive for like particles, whereas like particles repel in the case of all gauge forces. The Coulomb component in the case of the gauge forces determines the quantity known as the coupling constant.

(2) There appears to be an energy regime where the differences between the 3 gauge forces disappear and the coupling constants become equal. This Grand Unification energy is close to the Planck mass, which would link gravitational with gauge values. It is also possible that the character of the gauge forces would also become purely Coulombic at this energy.

So, we are entitled to propose a working hypothesis in which the gauge forces stem from a 3-component or ‘dimensional’ quantity charge, which is imaginary, and therefore quaternionic, in nature, to explain the difference in sign displayed by forces between like imaginary charges and like real masses. In addition, as Kant showed in the eighteenth century, inverse-square forces are direct manifestations of the 3-dimensionality of space, and, in a more modern perspective, are manifestations of the symmetry surrounding a point source. In this case, the parameters mass and charge could be manifestations of a quaternion structure, with three imaginary parts and one real, symmetric to the 4-vector structure of space and time, with three real parts and one imaginary.

space	time	charge	mass
$\mathbf{i}x\ \mathbf{j}y\ \mathbf{k}z$	it	$\boldsymbol{i}s\ \boldsymbol{j}e\ \boldsymbol{k}w$	$1m$

The 3-dimensional parts of the quaternions and 4-vectors, which are respectively irrotational and rotational, can be represented on mutually orthogonal axes, from which the fourth parts are excluded. The total structure has interesting parallels to an octonion and can be written in an octonion form, with the antiassociative aspects seemingly excluded from real physical manifestations.



The basic quaternion structure explains the coupling constant and the Coulombic term for each of the forces. But we still have to explain why the extra terms appear which make the 3 gauge interactions appear so different from each other at energies below Grand Unification. As we will show this is possible in a fundamental way, but it requires the interaction between *two* separate 3-dimensionalities, an interaction which explains why we have interacting point-particles at all.

The application of a 3-dimensional structure to charge as well as to space in the parameter group suggests that only 3-D concepts can be discrete, and that all discreteness in Nature comes from 3-dimensionality, as mass and time are in their different ways continuous. The discreteness of charge is obvious because it is a conserved quantity, but that of space is more subtle because it is nonconserved and so *has no fixed units*. It is, however, necessary to explain space as an observable quantity and also to explain Zeno's paradoxes.

It is, in fact, impossible to imagine a non-dimensional or continuous quantity as discrete. Dimensionality is needed to any method of division we can imagine, and to explain how we have zeros or crossover points. A discrete 'point' requires 3-D for its existence, and, as we will show, it requires 2 × 3-D. In addition, there is a natural relation between discreteness and anticommutativity which we see in physics. But, according to the universal rewrite system (Rowlands and Diaz, 2002, Marcer and Rowlands, 2014a,b, Rowlands, 2007, 2010, 2014), anticommutativity is also the origin of discreteness in mathematics in its creation of the idea of a repeating series of closed entities.

A double space

Suppose we set out the entire algebra and subalgebras of each of the four parameters in the Klein-4 group:

space	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}, i\mathbf{i}, i\mathbf{i}, i\mathbf{k}, i, 1$
		$\mathbf{i}, \mathbf{j}, \mathbf{k}, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, i, 1$
charge	quaternion (pseudovector)	$i\boldsymbol{i}, i\boldsymbol{j}, i\boldsymbol{k}, i, 1$
		$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, 1$
time	pseudoscalar (complex)	$i, 1$
mass	scalar	1

We see that immediately that charge, mass and time have algebras equivalent to the subalgebras of space. But if we put them altogether, they create an alternative algebra identical to that of Space.

combination	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}, i\mathbf{i}, i\mathbf{i}, i\mathbf{k}, i, 1$
		$\mathbf{i}, \mathbf{j}, \mathbf{k}, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, i, 1$
charge	quaternion (pseudovector)	$i\boldsymbol{i}, i\boldsymbol{j}, i\boldsymbol{k}, i, 1$
		$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, 1$
time	pseudoscalar (complex)	$i, 1$
mass	scalar	1

Since, in the group, they total to zero, we may see the combination as a kind of ‘antispace’ or alternative space to space itself. Note that its dimensionality comes from that of charge. If we assume these parameters are the only sources of physical knowledge, then the zero totality looks like a double space, whose two halves (with respective units represented, for convenience, in lower case symbols and capitals) mirror each other in some way.

space	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}, i\mathbf{i}, i\mathbf{i}, i\mathbf{k}, i, 1$
		$\mathbf{i}, \mathbf{j}, \mathbf{k}, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, i, 1$
combination	vector	$\mathbf{I}, \mathbf{J}, \mathbf{K}, i\mathbf{I}, i\mathbf{J}, i\mathbf{K}, i, 1$
		$\mathbf{I}, \mathbf{J}, \mathbf{K}, \boldsymbol{I}, \boldsymbol{J}, \boldsymbol{K}, i, 1$

Let us assume that these parameters are the whole of physics. How do we combine them? By combining the algebras.

Time	Space	Mass	Charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$
pseudoscalar	vector	scalar	quaternion

Working out every possible combination of the four requires 64 units. This turns out to be the algebra of the Dirac equation, the relativistic quantum mechanical equation of the fermion, the only true fundamental object that we know must exist. The result is another group, this time of order 64, rather than 4:

1	i				-1	$-i$			
$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{k}$	$\boldsymbol{j}$	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{k}$	$-\boldsymbol{j}$
$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{i}$	$\boldsymbol{k}$	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{j}$	$\boldsymbol{i}$	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$	$i\mathbf{k}$	$\mathbf{j}$	$-i\mathbf{i}\boldsymbol{i}$	$-i\mathbf{i}\boldsymbol{j}$	$-i\mathbf{i}\boldsymbol{k}$	$-i\mathbf{k}$	$-\mathbf{j}$
$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$	$i\mathbf{i}$	$\mathbf{k}$	$-i\mathbf{j}\boldsymbol{i}$	$-i\mathbf{j}\boldsymbol{j}$	$-i\mathbf{j}\boldsymbol{k}$	$-i\mathbf{i}$	$-\mathbf{k}$
$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{k}$	$i\mathbf{j}$	$\mathbf{i}$	$-i\mathbf{k}\boldsymbol{i}$	$-i\mathbf{k}\boldsymbol{j}$	$-i\mathbf{k}\boldsymbol{k}$	$-i\mathbf{j}$	$-\mathbf{i}$

We could also use the two ‘spaces’ as base units:

Space	Time-Mass-Charge
i j k	**I J K**
vector	vector

Here, we simply replace the quaternion units in the previous table (***i***, ***j***, ***k***) with the second set of vector units (**I**, **J**, **K**).

Groups do not need to be specified by all their elements. A small number of elements multiplied out can often generate the entire group. Here we only need 5 generators. 5 is a very significant number because it is the point in physics and mathematics at which symmetries become broken and chirality forces itself upon us. Thus, even though we could have generated the group from the six components **i**, **j**, **k**, **I**, **J**, **K**, or **i**, **j**, **k**, ***i***, ***j***, ***k***, which would have maintained perfect symmetry between the two ‘spaces’, the *minimum number of generators*, which is always what nature requires, forces us to break it.

The group can be structured as being made up of 4 complex number units and 12 sets of 5 generators, *any one of which*, for example,

$$\boldsymbol{i}\mathbf{i} \qquad \boldsymbol{i}\mathbf{j} \qquad \boldsymbol{i}\mathbf{k} \qquad i\boldsymbol{k} \qquad \boldsymbol{j}$$

could be used to produce the entire group. Even this does not exhaust the options. The 60 units could be organized as 12 sets of 5 generators in at least 64 different ways (with 4 independent options for sign changes, with changes of vectors and quaternions, and with real and imaginary units). The 60 is reminiscent of the 60 units of carbon in the C60 molecule, which are arranged in 12 pentagons, separated by hexagons which contain no extra atoms – in relation to the widespread appearance of structures of this type in biology and chemistry, as well as in physics, this is no coincidence. The symbols are, of course, arbitrary, and we could adopt any of the 64 × 12 sets of 5 available to us to represent physics, but it is most convenient to choose the set that is closest to our already accepted conventions.

The set of 5 generators is not unique (e.g. any row of the table of 64 can be used), but all sets follow the same pattern. Typically, beginning with

Time	Space	Mass	Charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$

we take one of each of the charge units ***i*** ***j*** ***k*** on to each of the units of the other three parameters, to create:

*i**k***	**i*i*** **j*i*** **k*i***	1***j***

The symmetry of one space **i** **j** **k** or the other ***i*** ***j*** ***k*** has to be broken.

Because space is a nonconserved quantity, and its component dimensions **i**, **j**, **k** can't be uniquely identified or distinguished from each other, the broken symmetry in physics becomes that of charge ***i***, ***j***, ***k***. (*Physics* requires this, though the symbols could be interchanged.) We can then show that 5 generators leads, among many other things, to the broken symmetry between the 3 charges which has troubled physics for more than forty years. This affects the nature of the charges as we observe them, and we can begin to recognise here the respective characteristics of weak, strong and electric charges, governed by the respective $SU(2)$, $SU(3)$ and $U(1)$ symmetries.

The charges adopt characteristics of the mathematical objects they are connected to. So we find that the charge we have represented by ***k*** becomes associated with a pseudoscalar quantity *i*; that the one represented by ***i*** becomes associated with three vector units **i**, **j** and **k**; and the one represented by ***j*** becomes associated with the scalar unit 1.

*i**k***	**i*i*** **j*i*** **k*i***	1***j***
weak	strong	electric
pseudoscalar	vector	scalar
$SU(2)$	$SU(3)$	$U(1)$

We see also that, though physics might require two 'spaces' for its specification, and that, though these two spaces may contain identical information, it presents itself differently within them, through a chirality:

*i***K**	**iI jI kI**	1**J**

And we see that the units of the second 'space' **I**, **J**, **K** now adopt the characteristics of the mathematical objects they are connected to:

pseudoscalar	vector	scalar

In addition to affecting charge, the combination also affects time, space and mass. We begin, once again, with

time	space	mass	charge
i	**i** **j** **k**	1	***i*** ***j*** ***k***

To create the generators, we distribute the charge units onto the others:

$$\boldsymbol{ik} \qquad\qquad \mathbf{i}\boldsymbol{i} \;\; \mathbf{j}\boldsymbol{i} \;\; \mathbf{k}\boldsymbol{i} \qquad\qquad 1\boldsymbol{j}$$

This creates *new* ‘compound’ (and ‘quantized’) physical quantities:

energy	momentum	rest mass
E	p_x p_y p_z	m

If we regard E, p_x, p_y, p_z, m simply as scalar values or ‘coefficients’, which are arbitrary in principle, and the algebraic operators $\boldsymbol{ik}$, $\mathbf{i}\boldsymbol{i}$, $\mathbf{j}\boldsymbol{i}$, $\mathbf{k}\boldsymbol{i}$,$1\boldsymbol{j}$ as defining the physical meaning, that is, the nature of the physical quantities involved, then we can package the whole information as

$$(\boldsymbol{ik}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)$$

In physics, this combined object is *nilpotent*, squaring to zero, because

$$(\boldsymbol{ik}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m) = 0 \qquad (1)$$

and we can identify this as Einstein’s relativistic energy equation

$$E^2 - p^2 - m^2 = 0$$

or, in its more usual form,

$$E^2 - p^2c^2 - m^2c^4 = 0$$

The nilpotent Dirac equation

The Dirac equation simply quantizes the nilpotent equation, using differentials in time and space applied to a phase factor for E and p. So equation (1) becomes

$$(-\boldsymbol{k}\partial / \partial t - i\boldsymbol{i}\nabla + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p.r})} = 0,$$

the Dirac equation for a free fermion, by simultaneously applying nonconservation and conservation. The operator, $(-\boldsymbol{k}\partial / \partial t - i\boldsymbol{i}\nabla + \boldsymbol{j}m)$, is like a coding of all the possible space and time variations, which is ‘decoded’ using a ‘phase factor’, here $e^{-i(Et - \mathbf{p.r})}$.

The most complete possible variation in space and time is defined by a phase factor which associates E with time and $\mathbf{p}$ with space, and then using the differentials $\partial / \partial t$ and ∇ to recover $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ from the phase factor. For a free particle, the most complete set of variations in space and time is given by $e^{-i(Et - \mathbf{p.r})}$, and the expression which will recover $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ using this as a phase factor is $(-\boldsymbol{k}\partial / \partial t - i\boldsymbol{i}\nabla + \boldsymbol{j}m)$. The phase factor $e^{-i(Et - \mathbf{p.r})}$ is then the minimum expression that the differentials $\partial / \partial t$ and ∇ can be applied to if space and time are to be varied without restriction.

Including all possible sign variations of E and $\mathbf{p}$, we obtain

$$(\mp \boldsymbol{k}\partial / \partial t \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m)\,(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p.r})} = 0$$

which is equivalent to a nilpotent Dirac equation of the form

$$(\mp \boldsymbol{k}\partial / \partial t \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m)\psi = 0.$$

We can also express it in operator form

$$(\pm\, i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\; e^{-i(Et - \mathbf{p.r})} = 0,$$

where the operators E and $\mathbf{p}$ become $i\partial / \partial t$ and $-i\nabla$ as in the usual canonical quantization.

The complete Dirac wavefunction in nilpotent form provides the particle state (represented by the first term) and the 3 possible states it could become by P, T and C transformations:

$$\begin{array}{ll} (i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m) & \\ (i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m) & P \\ (-\, i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m) & T \\ (-\, i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m) & C \end{array}$$

Parity, time reversal and charge conjugation are essentially reversals in the signs associated with space, time and charge.

We can interpret the expression

$$(\pm\, i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m) \rightarrow 0$$

as giving us both relativity and quantum mechanics. In quantum mechnaics we take the first bracket as an operator acting on a phase factor. The E and $\mathbf{p}$ terms can include any number of potentials or interactions with other particles. Squaring to 0 gives us the Pauli exclusion principle, because if any 2 particles are the same, their combination is 0.

Fermions appear to be point-like objects with norm 0. In effect, the creation of an object like

$$(\pm\, i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m),$$

squaring to zero, is the only way of defining a point in a space that is rotation and translation symmetric. We define a point at the interaction between two spaces, and the point-like fermion spends its whole existence switching between them (*zitterbewegung*), and we find that we cannot define a point, or the space in which it exists, without defining another space with which it interacts to produce a totality zero. One space will appear distorted with respect to the other, but each will contain the same information, or the information in one will be the reverse of the information in the other (Rowlands, 2007, 2013, 2014, 2015a).

Vacuum

To return to Pauli exclusion, and Nature as a totality of zero, we can imagine creating a particle (with all the potentials representing its interactions) in the form ($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*) and then being forced to structure the rest of the universe *or vacuum*, so that it can be represented by

$$-(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

So both the superposition and combination states of fermion and vacuum become zero:

$$(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) - (\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$
$$-(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$

The 'hole' left by creating the particle from nothing is the rest of the universe needed to maintain it in that state. We give it the name vacuum. So the vacuum for one particle cannot be the vacuum for any other. Vacuum tells us 'where' the other 'space', based on ***i***, ***j***, ***k***, is besides 'real' space, based on **i**, **j**, **k**. Its inaccessibility is demonstrated by the chirality between matter and antimatter, positive and negative energy states, and forward and backward directions in time.

The fermionic structure ($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*) posits two states with $+E$ and two with $-E$, which is equivalent to two with + time direction and two with –. And yet only $+E$ and $+t$ states are observed, and the universe is predominantly made of matter rather than antimatter. The second space, the one in which energy and time become negative, is the vacuum space, the one which encompasses the 'rest of the universe', as opposed to the real space defining the point-particle. It is the one in which charge is conjugated and time reversed simultaneously.

The nilpotent structure incorporates both on an equal basis, and it is interesting to recall that it was the seeming inability to do this which led Feynman to develop the path integral method of quantum mechanics as opposed to the previous use of Hamiltonian methods. Perhaps this indicates that, as long as we use the nilpotent formalism, we will be able to reformulate path integral calculations using Hamiltonians.

The nonlocal aspect of the fermionic nilpotent state ($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*) is defined by a continuous vacuum –($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*). The continuous vacuum, with its negative energy, appears to be that associated with gravity. We can consider the fermion itself, with positive energy and ($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*), as being in some sense an inertial term, and discrete fermions create their inertial mass through interaction with the continuous vacuum. Essentially, charge gives us the vacuum space dimensions, while mass gives it the continuous (negative) energy and nonlocality.

Further, we can use the operators ***k***, ***i***, ***j*** to effectively partition the continuous vacuum state, or the inertia which opposes this, into discrete components with a dimensional structure, which can then be identified as the weak, strong and electric components of vacuum, responding respectively to the discrete weak strong and electric charges. We can postmultiply $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ by the idempotent $\boldsymbol{k}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ any number of times, without changing its state

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \rightarrow (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

The idempotent acts as a vacuum operator, not changing the state. We can show that the same applies if we use the idempotents $\boldsymbol{i}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ and $\boldsymbol{j}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$:

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{i}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{i}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \rightarrow (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$
$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{j}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{j}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \rightarrow (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

In each case, every *alternate* bracket changes the sign of its E, $\mathbf{p}$ or E and $\mathbf{p}$ (equivalent to m) terms, leading to the creation of a bosonic state, here defined as a combination of fermion and antifermion, or equivalent. The only difference between the three is that the *alternate* brackets can be written as

$$\begin{array}{lll} (-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m) & \text{for} & \boldsymbol{k} \\ (\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m) & \text{for} & \boldsymbol{i} \\ (-\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m) & \text{for} & \boldsymbol{j} \end{array}$$

This means that they undergo respective T, P and C transformations. In addition the combinations with the unchanged bracket $(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$ means that the three operators produce different types of bosonic states, respectively:

spin 1
spin 0
paired fermion

We can regard T and C as transformations in vacuum space because they concern vacuum space quantities and lead to negative energy states. The P transformation concerns the real space quantity and leads to a positive energy state. However, it could occur in vacuum space by a combined TC transformation, and so real fermion / antifermion states are not purely left- / right-handed, as their weak interactions would suggest. The degree of overlap is determined by the m term, which is said to be the product of the *zitterbewegung*, or switching of a fermion between + and $-E$ states or real space and vacuum.

We can see how the 3 bosonic states are related to vacua produced by the 3 quaternionic operators, and the 3 discrete transformations:

$$\begin{array}{c} \text{weak} \qquad\qquad \text{spin 1} \qquad\qquad\qquad T \\ (\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}\, (\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}\, (\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}\, (\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m) \ldots \\ (\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\, (-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\, (\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\, (-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m) \ldots \end{array}$$

electric spin 0 C

$$(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{j}\, (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{j}\, (\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{j}\; i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m) \ldots$$
$$(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, (-\, i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, (-\, i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m) \ldots$$

strong paired fermion state P

$$(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{i}\, (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{i}\, (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{i}\, (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m) \ldots$$
$$(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, (i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, (i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m) \ldots$$

Vacuum space and charge

Now, the vacuum space, is the one defined by the units connected with charges, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$. Here we see that the units have multiple roles:

Charges
PCT operators
Dimensions of 'vacuum space'
Generators of the 3 additional terms in the Dirac wavefunction
Creation operators converting fermions to 3 types of boson

The complete vacuum, defined by $(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$ or $-(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$, can be considered as equivalent to that defined by gravity or inertia, which the gauge forces split into 3 dimensional components. Also, taking $(i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$ to convey the angular momentum information about a particle state, we can see that the units also say something about the respective conservation laws of handedness, direction and magnitude of this quantity. Ultimately, this introduces the *SU*(2), *SU*(3) and *U*(1) group operators into particle physics, and explains why the symmetry between the 3 charges is broken. But the key idea is that the total angular momentum information can be obtained either through $\boldsymbol{k}$, $\boldsymbol{i}$ and $\boldsymbol{j}$ of iE, $\mathbf{p}$ and m, or through the $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ of p.

The two 'spaces' contain *exactly equivalent information about the whole of physics*, although in one case the symmetry between the units appears to be broken and in the other it is exactly preserved. To show that the information is equivalent, we will consider how the uniqueness of a fermion wavefunction is determined to maintain Pauli exclusion. So if we know the iE, $\mathbf{p}$ and m values in $(\pm\, i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$ for any fermion, we can show that those of another fermion must be different. But, there is also a completely different way of establishing Pauli exclusion, that is, by realising that fermions have antisymmetric wavefunctions.

The nilpotent structure explains immediately why we have Pauli exclusion between fermions, but the alternative, *conventional*, way of explaining this property leads us to a profound insight on the nature of the information available in quantum systems if we structure it in nilpotent form. We define fermion wavefunctions to be antisymmetric, so that:

$$(\psi_1\psi_2 - \psi_2\psi_1) = -(\psi_2\psi_1 - \psi_1\psi_2)$$

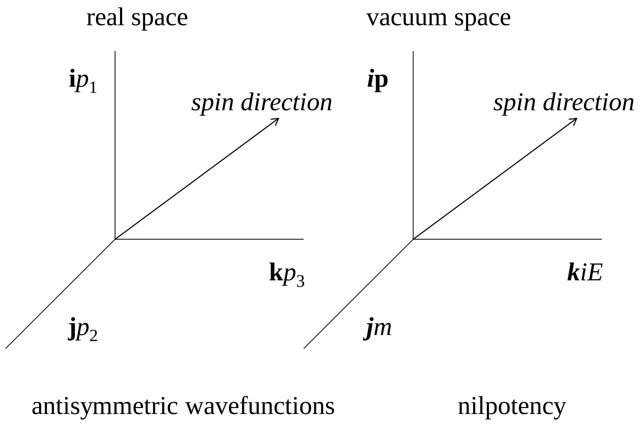
In nilpotent terms, we write $(\psi_1\psi_2 - \psi_2\psi_1)$ as

$$(\pm\, i\boldsymbol{k}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\,(\pm\, i\boldsymbol{k}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$$
$$-\,(\pm\, i\boldsymbol{k}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\,(\pm\, i\boldsymbol{k}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$$
$$= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8i\mathbf{p}_1 \times \mathbf{p}_2 = -\,8i\mathbf{p}_2 \times \mathbf{p}_1.$$

This result is clearly antisymmetric, but it also has a quite astonishing consequence, for it requires any nilpotent wavefunction to have a **p** vector, in *real space*, the one defined by the axes **i**, **j**, **k**, at a *different orientation* to any other. The wavefunctions of all nilpotent fermions then instantaneously correlate because the planes of their **p** vector directions must all intersect. At the same time, the nilpotent condition requires the E, **p** and m combinations to be unique, and we can visualize this as constituting a unique direction in *vacuum* space along a set of axes defined by $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$ or **k**, **i**, **j**, with coordinates defined by the values of E, **p** and m.

The directions of the vectors in each space carry *all the information* available to a fermionic state, and so the information in the two spaces is totally dual, and is equivalent to the instantaneous direction of the spin in the real space. The total information determining the behaviour of a fermion and even of the entire universe is contained in a single spin direction. The information here must be the same as in the combination of iE, p and m. That is, we can represent the unique spin direction in parallel axes in two different spaces.

Pauli exclusion



Though the duality results in fermion and vacuum occupying separate 3-dimensional ‘spaces’, which are combined in the double Clifford algebra defining the singularity state, these ‘spaces’, though seemingly different in observational terms, are truly dual, each containing the same information (angular momentum), and the duality manifests itself directly in many physical forms.

Angular momentum, which in some form combines all the information of the phase space of the particle, shows this duality directly, because, as a 3-dimensional pseudovector quantity, it has the rotation symmetric and nonconserved 3-dimensionality of space; but, at the same time, it shows 3 different aspects and 3 different conservation laws connected with them, which are connected by a different and rotation asymmetric 3-dimensionality.

One of the remarkable aspects of this analysis is that it shows that the connection between space and time, in special relativity, is not really 4-dimensional at all, but *3-dimensional,* though the 3 dimensions involved are different from those of space. We can write out the relativistic nilpotent as

$$\boldsymbol{ik}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m$$

or even

$$\boldsymbol{ik}t + \mathbf{i}\boldsymbol{i}x + \mathbf{j}\boldsymbol{i}y + \mathbf{k}\boldsymbol{i}z + \boldsymbol{j}\tau$$

The 3 components of space and momentum are denoted by the 3 blue vector coefficients, but they do not add directly to the pseudoscalar *iE* or *it*. They only connect by additional, red, coefficients, which are those of another space. Also, the space-time connection is not privileged over those with mass and charge. If we write the relativistic nilpotents in the form:

$$\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$$

and

$$\boldsymbol{ik}t + \boldsymbol{i}\mathbf{i}\mathbf{r} + \boldsymbol{j}\tau$$

we can see that there is an immediate analogy with the cases of 3 static forces in equilibrium or a closed path round a right-angled triangle (say with lengths 3, 4 and 5) where

$$\boldsymbol{ik}W + \mathbf{i}T_1 + \boldsymbol{j}T_1$$

or

$$\boldsymbol{ik}5 + \boldsymbol{i}4 + \boldsymbol{j}3$$

Where we have a zero total of this kind, we can consider it to be a case of finding the resultant in the dual space which cancels that in real space, showing the significance of Newton's third law in this connection.

If we represent particles in terms of their charge structures, this can be done in two different spaces. If we look at baryons composed of 3 quarks, we can see their behaviour in terms of real space, or that of the momentum operator, and this has rotation symmetry. But we can also see it in terms of vacuum or charge space, and this is rotation asymmetric.

Space and charge show us the dual aspects of angular momentum, nonconserved and symmetric, and conserved and asymmetric. All 3-dimensional quantities are of this kind, one that is ultimately expressed in Noether's theorem. To any 3-dimensionality that gives us nonconservation and rotation symmetry, there is always attached another that gives the same information using conservation and rotation asymmetry.

This is why the ultimate expression of physics requires phase space. To any description involving space or space-time, we require an equivalent description involving vacuum space, manifesting itself through charge, or through the combination of charge and space and time, which is described as energy-momentum. At the quantum (point-particle) level, these cannot be separated. This is how the combination of all 3 charges becomes necessary to describe the behaviour of the particular charges which is associated with spatial 3-dimensionality. The behaviour of the strong interaction can be completely determined by the spatial 3-

dimensionality of the momentum operator in the nilpotent wavefunction. However, the situation of the strong charge as a 3-dimensional operator within the fermion state requires the application of the other 3-dimensionality associated with charge.

Baryons

Effectively, the vector aspect of the strong charge requires a source term and corresponding vacuum with three components. Though we clearly cannot combine three components in the form:

$$(i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

as this will automatically reduce to zero, we can imagine a three-component structure in which the vector nature of **p** plays an explicit role

$$(i\boldsymbol{k}E \pm \boldsymbol{i}\,\mathbf{i}p_x + \boldsymbol{j}m)\,(i\boldsymbol{k}E \pm \boldsymbol{i}\,\mathbf{j}p_y + \boldsymbol{j}m)\,(i\boldsymbol{k}E \pm \boldsymbol{i}\,\mathbf{k}p_z + \boldsymbol{j}m)$$

This has nilpotent solutions when $\mathbf{p} = \pm\, \boldsymbol{i}\,\mathbf{i}p_x$, $\mathbf{p} = \pm\, \boldsymbol{i}\,\mathbf{j}p_y$, or $\mathbf{p} = \pm\, \boldsymbol{i}\,\mathbf{k}p_z$, or when the momentum is directed entirely along the x, y, or z axes, in either direction, though these, of course, are arbitrarily defined. Any other phases can be written as a superposition of these. Using the appropriate normalization, these reduce to

$$\begin{array}{ll}
(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m) & +RGB \\
(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m) & -RBG \\
(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m) & +BRG \\
(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m) & -GRB \\
(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & +GBR \\
(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & -BGR
\end{array}$$

with the third and fourth changing, very significantly, the sign of the **p** component. Because of this, there has to be a maximal superposition of left- and right-handed components, thus explaining the zero observed chirality in the interaction (and the mass of the baryon).

The group structure required to maintain these phases is an $SU(3)$ structure, with eight generators and a wavefunction, exactly as in the conventional model using coloured quarks,

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB).$$

'Colour' transitions in the 3-component structures are produced either by an exchange of the components of **p** between the individual quarks or baryon components, or by a relative switching of the component positions, independently of any real distance between the components. No direction can be privileged, so the transition must be gauge invariant, and the mediators must be massless, exactly as with the eight massless gluons of the gluon structure. We have written only the first term of the 4-component spinors, but we have retained the two spin states, as these will be needed explicitly.

The complete wavefunction will, in effect, contain information from the equivalent of six allowed independent nonlocally gauge invariant phases, all existing simultaneously and subject to continual transitions at a constant rate:

$$
\begin{array}{ll}
(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \ \dots\ + \boldsymbol{j}m)\ \ (i\boldsymbol{k}E + \ \dots\ + \boldsymbol{j}m) & +RGB \\
(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \ \dots\ + \boldsymbol{j}m) & -RBG \\
(i\boldsymbol{k}E + \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \ \dots\ + \boldsymbol{j}m) & +BRG \\
(i\boldsymbol{k}E - \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \ \dots\ + \boldsymbol{j}m) & -GRB \\
(i\boldsymbol{k}E + \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & +GBR \\
(i\boldsymbol{k}E - \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \ \dots\ + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & -BGR
\end{array}
$$

The simultaneous existence of all phases further means that *individual* quarks, and such identifying characteristics as electric charges, are not identifiable by their spatial positions (unlike, say, the proton and electron constituting a hydrogen atom), thus explaining, for example, why the neutron has no electric dipole moment. Just as $U(1)$ establishes that spherical symmetry of a point source requires the rotation to be performed independently of the length of the radius vector, so $SU(3)$ requires the rotation to be independent of the coordinate system used. In terms of Noether's theorem, while $U(1)$ conserves the magnitude of angular momentum, $SU(3)$ conserves the direction.

These are the established characteristics of the strong interaction and here we have an explanation, derived on an analytic basis, for a force with these characteristics. If we now look at how the expression

$$(i\boldsymbol{k}E \pm \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}m)\ (i\boldsymbol{k}E \pm \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}m)\ (i\boldsymbol{k}E \pm \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}m)$$

with its spatial 3-dimensionality, fits into the 3-dimensionality associated with the charge picture, we can recall that the 4 components in the Dirac spinor, can be seen to represent the fermion as seen in terms of gravity / inertia, strong, weak and electric vacua. The 3 momentum components have to be mapped onto the space created by the 3 charges. The principal options are:

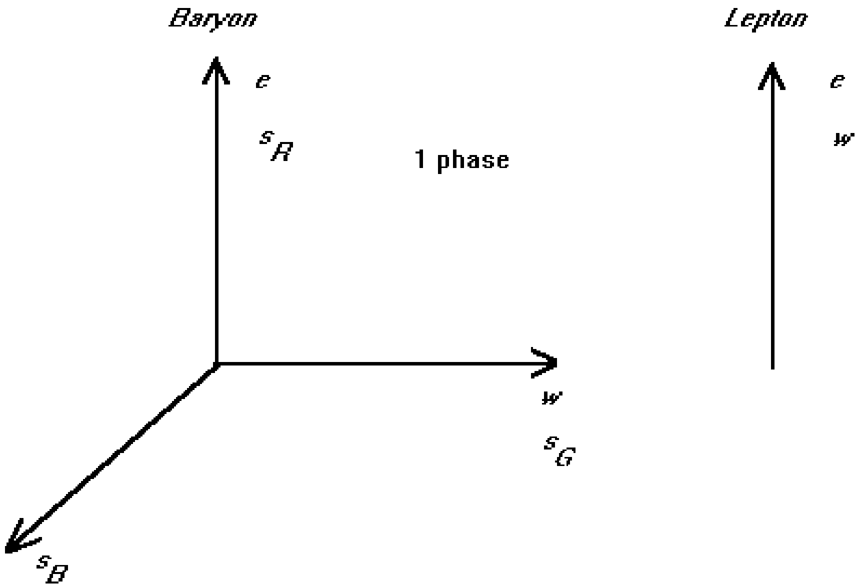
$$
\begin{array}{r}
\textit{inertial} \\ \textit{strong} \\ \textit{weak} \\ \textit{electric}
\end{array}
\quad
\begin{pmatrix}
i\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\
i\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\
-i\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\
-i\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m
\end{pmatrix}
\begin{pmatrix}
i\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\
i\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\
-i\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\
-i\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m
\end{pmatrix}
\begin{pmatrix}
i\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\
i\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\
-i\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\
-i\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m
\end{pmatrix}
$$

and

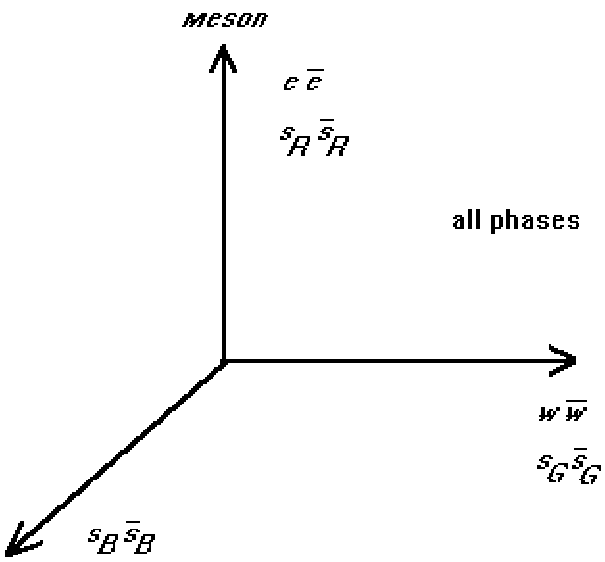
$$
\begin{array}{r}
\textit{inertial} \\ \textit{strong} \\ \textit{weak} \\ \textit{electric}
\end{array}
\quad
\begin{pmatrix}
ik\mathbf{E} \pm i\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\
ik\mathbf{E} \mp i\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\
-ik\mathbf{E} \pm i\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\
-ik\mathbf{E} \mp i\boldsymbol{\sigma}.\mathbf{p}_3 + jm
\end{pmatrix}
\begin{pmatrix}
ikE \pm i\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\
ikE \mp i\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\
-ikE \pm i\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\
-ikE \mp i\boldsymbol{\sigma}.\mathbf{p}_2 + jm
\end{pmatrix}
\begin{pmatrix}
ik\mathbf{E} \pm i\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\
ik\mathbf{E} \mp i\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\
-ik\mathbf{E} \pm i\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\
-ik\mathbf{E} \mp i\boldsymbol{\sigma}.\mathbf{p}_1 + jm
\end{pmatrix}
$$

Here, we assume only one dimension of **p** is nonzero, say $\mathbf{p}_3$, so determining the charge structure, with the 3 charges distributed.

We can also represent it using a phasor diagrams, with the charges separated in baryons, but not in leptons:



For mesons we would have:



If the strong charge is absent, we need only one phase, with all charges aligned.

$$
\begin{array}{r}
\textit{inertial} \\
\textit{strong} \\
\textit{weak} \\
\textit{electric}
\end{array}
\quad
\begin{pmatrix}
\boldsymbol{ik}E \pm \boldsymbol{i\sigma}.\mathbf{p} + \boldsymbol{j}m \\
\boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p} + \boldsymbol{j}m \\
-\boldsymbol{ik}E \pm \boldsymbol{i\sigma}.\mathbf{p} + \boldsymbol{j}m \\
-\boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p} + \boldsymbol{j}m
\end{pmatrix}
$$

The wavefunctions suggest that the underlying charge structures of quarks are as in the first table below, with the more familiar fractional electric charges appearing in observation, as in the second table, because the strong interaction is absolutely gauge invariant:

	Blue	**Green**	**Red**
up	e	e	0
	B	0	0
down	0	0	$-e$
	B	0	0

	Blue	**Green**	**Red**
up	$2e / 3$	$2e / 3$	$2e / 3$
	$B / 3$	$B / 3$	$B / 3$
down	$-e / 3$	$-e / 3$	$-e / 3$
	$B / 3$	$B / 3$	$B / 3$

A similar fractionalization of electric charge occurs in the fractional quantum Hall effect, where the fractional status of the charge on an electron is produced by its *weak* interaction with an odd number of flux lines > 1. In both cases, it is a different electric-charge independent force which ensures that the electric charges become fractional. The underlying charge structures can be used to represent the Standard Model simply in a set of 4 quark-lepton tables. They also predict Grand Unification of the 4 forces at the Planck mass, explain *CP* violation, solve a serious anomaly in the Higgs mechanism as applied to finding the masses of fermion states, etc. (Rowlands, 2007, 2014, Rowlands and Cullerne, 2001)

A

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
d	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
s	$+e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$-w$	$Z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
b	$+e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$Z_T\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$

B

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
d	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
s	$+e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$-w$	$Z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
b	$+e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$-w$	$Z_T\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$

C

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$0\mathbf{i}$	$1\mathbf{k}$	$0\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
d	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{i}$	$1\mathbf{k}$	$0\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$0\mathbf{i}$	$1\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
s	$+e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{i}$	$1\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$0\mathbf{i}$	$1\mathbf{k}$	$0\mathbf{j}$
	$-w$	$Z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
b	$+e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$0\mathbf{i}$	$1\mathbf{k}$	$0\mathbf{j}$

L

		$\bar{e}$	$\bar{e}$	ν_e
	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{j}$
	$+s$	$0\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{i}$
	$+w$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{k}$
				e
	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$0\mathbf{i}$
	$+w$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{k}$
		$\bar{\mu}$	$\bar{\mu}$	ν_μ
	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{j}$
	$+s$	$0\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{i}$
	$-w$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{k}$
				μ
	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$1\mathbf{i}$	$0\mathbf{i}$
	$-w$	$0\mathbf{i}$	$0\mathbf{k}$	$z_P\mathbf{k}$
		$\bar{\tau}$	$\bar{\tau}$	ν_τ
	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{j}$
	$+s$	$0\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{i}$

3-dimensionalities deriving from space and charge

We have seen that 3 components of charge is a 3-dimensionality separate from that of space, but acting against it to produce zero totality. Other 3 dimensionalities emerge from those of space and charge. So, as we have seen, we have 3 quarks in a baryon because we have 3 dimensions of space and also of momentum. It is simultaneously an expression of the existence of 3 types of charge.

The existence of 3 generations of fundamental particles is an interesting case, which comes from *CPT* symmetry. According to the Dirac equation, massless fermions, if they existed, would be purely left-handed. So the measure of a fermion is a measure of how much right-handedness it contains. Higher mass fermions are more right-handed than lower mass ones. We can see the second and third generations as producing step-functions in right-handedness and so in mass values by successively violating *P* and *T* symmetries. The relative masses of the quarks and leptons in the 3 generations seem to be determined by the electroweak and strong coupling constants. This is discussed in the Appendix.

Now, a nonconserved 3-D quantity has rotation symmetry, so the parts are indistinguishable. This is characteristic of quantities whose dimensionalities come from that of space. A conserved 3-D quantity has rotation asymmetry, with the parts distinguishable and different, though in a systematic way which comes from its interaction with its counterpart in real space. These can be related to vacuum space derived from the units $\boldsymbol{i}$ $\boldsymbol{j}$ $\boldsymbol{k}$, and we only know about these indirectly through the combination.

There is, finally, an interesting application of these ideas in a seemingly 'pure' mathematical context, which gives indications of actually being determined by a physical argument. It has been a long-held belief of mine that physics and mathematics are linked at a much deeper level than one merely 'using' the other, and that they exhibit same basic patterns and structures. Several examples can be suggested, but one is of special interest because it leads to an iconic pure mathematical equation. Here, we wish to find a *physical* answer to a specifically mathematical question: if we start with a scalar unit real number, say 1, how do we find the conjugate number to obtain totality zero? Physically, *mass*, which uses scalar, real numbers, is a conserved, real, and nondimensional quantity. To find the conjugate to this, we look for something that is nonconserved, imaginary and dimensional.

In mathematics and physics, nonconservation is expressed by differentiation: e is a number that arises only out of differentiation; i is the basis of imaginary numbers; and π is a number that arises only out of 3-dimensionality. So $e^{i\pi}$ gives us a number which is conjugate to 1. The transformations here are equivalent to the physical *TCP* or *PTC*, and 3-dimensionality, as always, plays a doubly crucial role. So the famous equation $e^{i\pi} = -1$ or $e^{i\pi} + 1 = 0$ may point to a direct connection between mathematics and physics at the most fundamental level.

Appendix: 3 generations of fermions and their masses

The 3 generations of fundamental fermions represent a special aspect of 3-dimensionality connected with *C*, *P* and *T* and *m*, *p* and *E*.

u	d	ν_e	e
c	s	ν_μ	μ
t	b	ν_τ	τ

They have ascending masses based on the symmetry-breaking associated successively with violations of parity and time-reversal symmetry, which are the result of electroweak couplings, whereas differences between quarks and leptons in the same generation and isospin state appear to be due to the presence of strong couplings in the former and their absence in the latter. In terms of mass, the highest level, with all symmetries broken, is the maximal value associated with the Higgs field, the source of all mass. This is seen in the third generation. The second generation has masses associated with the energy scale of the regular weak interaction, where symmetry-breaking first occurs and below that associated with the breaking of all symmetries. This can be taken as close to the mass of the Z boson (91.2 GeV).

The masses of the particles in the higher generations, in particular, are related to the creation of an electric charge unit (which is generally connected to the creation of electroweak mass as it generates a degree of right-handedness not strongly present in the pure weak interaction) within a distance of the order of the Compton wavelength, either standard or reduced. The standard Compton wavelength is the length scale relevant to mass conversions into energy, where the reduced wavelength is the one used when discrete mass is created.

The mass of t, the heaviest fermion, is close to maximal coupling to the Higgs field, expectation value $f \approx 246$ GeV. So the mass of t becomes $\approx f / \sqrt{2} \approx 174$ GeV, with no specific process needed to explain it. In the same generation, but with no strong charge, τ has a mass close to that which a single electric charge e would need to be confined by energy conservation to the reduced Compton radius associated with 246 GeV. This works out at $\approx f\alpha$, where α is the electromagnetic fine structure constant. The interactions associated with τ (unlike those associated with t) are purely electroweak, and the weak interaction does not extend to this length. The mass of b should be connected to that of its isospin 'down' partner, τ, and, in fact, it appears to be equivalent to that of τ divided by α_3, the strong interaction coupling at the associated energy (approximately 2.35).

In the second generation, with one degree less symmetry-breaking, masses are reduced by the appropriate electroweak factor, in this case α, and c has a mass equivalent to that of t times α, or $f\alpha / \sqrt{2}$. This is also equivalent to that producing the charge e at the reduced Compton wavelength generated by the mass of maximal coupling to the Higgs field (i.e. that of t). The c mass (at 1.2 to 1.3 GeV) differs from that of t due to the electroweak coupling only, as the strong coupling is unchanged. The muon μ, the counterpart in this generation to τ, has the mass needed to generate e over the standard Compton wavelength for the Z energy scale. (We may note here that the energy scale here is determined by that of a real particle, whereas that for t is determined by the energy of a field.) As in the third generation, s, the isospin 'down' counterpart to μ, can again be taken to be equivalent to μ with an additional strong interaction, and so has a mass equivalent to that of μ divided by α_3, the strong interaction coupling at the associated energy (which, in this case, is approximately 0.9).

In the first generation, where there is no electroweak symmetry breaking, the masses are again reduced by a factor close to α. Here, the masses of the u and d quarks are not well established, but that of u is certain of the order of the mass of c times α at 9 MeV (although α may conceivably determine the mass ratios of the whole generations rather than just the isospin 'up' component). The electron mass is close to $2\alpha / 3$ times that of μ though the reason for the factor 2/3 is not readily apparent. (See the suggestions in Rowlands, 2015b.) The strong interaction does not dominate at this level in the mass of the *d* quark where its contribution would be a relatively small mass of e / α_3 (where α_3 would be a number considerably greater than 1). Clearly, the main contribution at this scale would be electroweak.

References

Hestenes, D. 1966, *Space-Time Algebras, Gordon and Breach*, New York

Marcer, P. J. and Rowlands, P., 2014a, Information, Bifurcation and Entropy in the Universal Rewrite System, *International Journal of Computing Anticipatory Systems*, 27, 203-215

Marcer, P. J. and Rowlands, P., 2014b, Universal Rewrite and Self-Organization, *International Journal of Computing Anticipatory Systems*, 27, 227-246

Rowlands, P., 1983, The fundamental parameters of physics. *Speculat. Sci. Tech.*, **6**, 69-80

Rowlands, P., 1991, *The Fundamental Parameters of Physics: An Approach towards a Unified Theory*, PD Publications, Liverpool

Rowlands, P., 2001, A foundational approach to physics, arXiv:physics/0106054

Rowlands, P., 2003, Representations of a Fundamental Theory, in K. Bowden (ed.), *Boundaries*, 18-47

Rowlands, P. 2007, *Zero to Infinity The Foundations of Physics*, Singapore, London and Hackensack, NJ: World Scientific

Rowlands, P. 2009, Are there alternatives to our present theories of physical reality?, arXiv:0912.3433

Rowlands, P., 2010, Mathematics and Physics as Emergent Aspects of a Universal Rewrite System, *International Journal of Computing Anticipatory Systems*, **25**, 115-131

Rowlands, P., 2013, Space and Antispace, in R. L. Amoroso, L. H. Kauffman and P. Rowlands (eds.), *The Physics of Reality Space, Time, Matter, Cosmos*, World Scientific, 29-37

Rowlands, P. 2014, *The Foundations of Physical Law*, Singapore, London and Hackensack, NJ: World Scientific

Rowlands, P. 2015a, *How Schrödinger's Cat Escaped the Box*, Singapore, London and Hackensack, NJ: World Scientific

Rowlands, P. 2015b, Reducing the Number of Free Parameters, in Grenville J. Croll and Nicky Graves Gregory (eds.), *Explorations*, 223-259

Rowlands, P. 2015c, How many dimensions are there?, in Richard L Amoroso, Louis H. Kauffman and Peter Rowlands (eds.), *Unified Field Mechanics Natural Science Beyond the Veil of Spacetime Proceedings of the IX Symposium Honoring Noted French Mathematical Physicist Jean-Pierre Vigier*, World Scientific, 2015, 46-54

Rowlands, P. and Cullerne, J. P., 2001, The connection between the Han-Nambu quark theory, the Dirac equation and fundamental symmetries. *Nuclear Physics* **A 684**, 713-5

Rowlands, P. and Diaz, B. 2002, A universal alphabet and rewrite system, arXiv:cs.OH/0209026